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MacMURCHY

ARITHMETIC

& BROWN

FOR JUNIOR PUPILS.

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We have the pleasure of sending you a copy of our NEW ARITHMETIC FOR JUNIOR PUPILS, which we shall be glad if you will, as you have leisure, carefully examine. Any remarks upon it or suggestions you may be disposed to make we shall be pleased to receive.

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Canadian Series of School Books.

ARITHMETIC

FOR

JUNIOR PUPILS.

BY

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In this ELEMENTARY ARITHMETIC, care has been taken to supply the wants of the Junior Pupils in the Schools of the Dominion.

Compound Quantities have been introduced immediately after the Simple Rules in deference to the opinion of the Public School Teachers, as expressed by resolution adopted in Annual Convention at Toronto, August, 1882.

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ARITHMETIC.

SECTION I.

1. **Arithmetic** is that part of learning which shews how to use numbers.

2. **Unit.** A unit means one. Any single thing, as a pencil, a cent, a boy, may be taken as a unit.

3. **Quantity.** A unit, or a group of units, is called a quantity; concrete, if the kind of unit is named; abstract, if the unit is not named; five apples, ten cents, twelve boys are concrete quantities; five, ten and twelve are abstract.

4. **Number** shews the size of a quantity by telling how many units it contains: thus in eight apples, six sheep, four dollars, eight, six and four are numbers.

5. **Measure.** The number of units in a quantity is often called its measure. The same quantity may have different measures according to the unit taken; thus the same quantity of money may be called one dollar, four quarters, one hundred cents. One, four and one hundred are measures.

6. **Integers.** The common numbers one, two, three, etc., are called integers, or whole numbers.

7. **Cardinal and Ordinal.** The common numbers, one, two, three, etc., are called cardinal numbers to distinguish them from first, second, third, etc., which mark the order in which any thing is taken, and are called ordinal numbers.

8. Numbers are expressed by words, by marks called figures, or by letters. Example: one, two, three, etc.; 1, 2, 3, etc.; I, II, III, etc.

9. Notation is the art of writing numbers by figures, letters or other marks.

10. Numeration is the art of reading or writing in words numbers written in figures or letters.

11. Arabic Notation. In this method the first nine numbers are denoted as follows:

One, Two, Three, Four, Five, Six, Seven, Eight, Nine.
1, 2, 3, 4, 5, 6, 7, 8, 9.

These are called significant figures.

The figure 0 (nought) means none, and is used to denot the absence of number.

It is called a cipher or zero.

By means of these ten figures all numbers can b denoted by the following method:

Local Value. The value of any figure is increase ten-fold for every place that it is moved to the left b putting another figure to the right hand of it.

Thus we have,

Ten, Twenty, Thirty, Forty, Fifty, Sixty, Seventy,
10, 20, 30, 40, 50, 60, 70,

Eighty, Ninety.
80, 90.

Again, moving the significant figures another place t the left, we have,

One hundred, Two hundred, Three hundred,
100, 200, 300,

Four hundred,	Five hundred,	Six hundred,
400,	500,	600,
Seven hundred,	Eight hundred,	Nine hundred.
700,	800,	900.

The 0's are used in the absence of significant figures: thus, eleven (ten and one) is 11, twelve (ten and two) is 12, thirteen (ten and three) is 13, fourteen (ten and four) is 14, and so on up to nineteen (ten and nine), which is 19.

So twenty-one is 21, twenty-two is 22, twenty-three is 23, and so on up to ninety-nine, which is 99.

Again, a hundred and one is 101, a hundred and two is 102, a hundred and twenty is 120, a hundred and thirty-five is 135, two hundred and seventy-three is 273, and so on up to nine hundred and ninety-nine—999, which is the largest number denoted by three figures.

EXERCISE I.

1. Read or write in words, 56, 78, 83, 95, 70, 49, 26, 34, 67.

2. Express in Arabic notation, Twenty-nine, Eighteen, Thirty-six, Forty-eight, Fifty-four, Sixty-five, Seventy-six, Eighty-nine, Ninety-two.

3. Read or write in words, 108, 110, 115, 118, 124, 136, 205, 210, 212, 316, 340, 465, 587, 609, 728, 888, 964.

4. Express in Arabic notation, One hundred and six, Two hundred and ten, Three hundred and forty, Four hundred and sixty-five, Five hundred and four, Six hundred and seventy-eight, Seven hundred and forty-two, Eight hundred and sixteen, Nine hundred and eighty-seven.

---

12. We have seen that the figures in the first three places, counting from right to left, denote respectively units, tens of units and hundreds of units, or, briefly, units, tens and hundreds. Figures in the next three places, viz., the fourth, fifth and sixth, denote respectively thousands, tens of thousands and hundreds of thousands.

Thus we have,

One thousand..........	1000	or	1,000
Ten thousand..........	10000	or	10,000
One hundred thousand..	100000	or	100,000

As the names of the second three figures are the same as those of the first three, except that we have thousands instead of units, it is useful to separate the second period from the first by a comma; we can then write or read the two periods separately. Thus, to read in words 5608 or 5,608 we say five thousand (for the second period) six hundred and eight (for the first).

So, to write in figures Twelve thousand four hundred and sixty, we have 12 for the second period and 460 for the first, or altogether, 12,460.

Again, to read 608017, or 608,017, we have Six hundred and eight thousand and seventeen units.

Note. The s at the end of hundreds, thousands and so on, and the word units at the close of the number, are usually omitted.

To write in figures Four hundred and eighty thousand nine hundred and six, we have for the second period 480 and for the first 906, or altogether, 480,906.

EXERCISE II.

1. Read or write in words 1206, 1357, 2615, 3008, 4600, 5780, 6000, 7812, 8579, 9999.

2. Write in figures, One thousand four hundred, Two thousand six hundred and fifty, Three thousand and forty-

eight, Four thousand, Five thousand seven hundred and nine, Six thousand two hundred and fifty-four, Seven thousand, Eight thousand and ninety, Nine thousand and nine.

3. Write in words, 16400, 35008, 70463, 85000, 94600, 105634, 368000, 400980, 500600, 643578, 750860, 800008, 909099.

4. Write in words, 3568, 4004, 7089, 18600, 48065, 82020, 95095, 160840, 207536, 315000, 600028, 701111, 854900, 999999, 909090.

5. Write in figures, Seventy-one thousand four hundred and thirty-nine, Four hundred and five thousand six hundred and seven, One hundred thousand one hundred, Eight hundred and eighty thousand eight hundred and eighty, One hundred and forty-four thousand, Nine hundred thousand nine hundred and ninety, Eighty thousand and eight, Five hundred and sixty thousand and seventy.

---

13. As there is little use in occupying the time of beginners with numbers of more than six or seven figures, we will merely mention that the three figures in the third period denote millions, tens of millions and hundreds of millions ; the three figures in the fourth period denote billions, tens of billions and hundreds of billions, and so on with quadrillions, quintillions, etc.

14. To write large numbers in figures, the beginner will find it useful to set down a number of periods filled with 0's and having the initial of its name over each period, thus,

B. M. T. U.
000,000,000,000.

Keeping this before him as a model, he has merely to write the proper figures in each period.

Example 1. To write Five million sixty thousand eight hundred :

5 in the third period, 060 in the second and 800 in the first.

Thus he will have 5,060,800.

Note. The 0's to the left of the significant figure 5 in the highest period are omitted, but not in any of the lower periods.

Example 2. Write in figures forty million and sixty-five.

40 in the third period ......
000 in the second period......
065 in the first period ....
} or altogether 40,000,065.

15. To read in words any large number written in figures, divide it by commas into periods of three figures, beginning from the right, and then, starting with the highest, read the figures in each period just as if they stood alone, and add the name of the period.

Example 1. Read in words 6012856.

This when divided is 6,012,856.

For the third period Six million ; for the second period twelve thousand ; for the first period eight hundred and fifty-six.

Example 2. Read in words, 70508900.

When divided it is 70,508,900.

Seventy million five hundred and eight thousand nine hundred.

Example 3. 800080008, when divided, is 800,080,008 :

Eight hundred million eighty thousand and eight.

Example 4. 13000078, when divided, is 13,000,078 :

Thirteen million and seventy-eight.

Note. It should be observed, as in the above example, that when a period contains 0's only it is omitted in the reading, but the 0's cannot be omitted in writing the number.

16. Digits. The figures which denote a number are called its digits.

Thus, 76 is a number of two digits, and 185 is a number of three digits.

EXERCISE III.

1. Write in words, 7000000, 25064780, 65408700, 812004005, 7500650000.

2. Write in figures, Five million six thousand and seven, Fourteen million eighty-five thousand and sixteen, Nine hundred million eight hundred thousand and seven hundred, Three billion twenty million one thousand and ninety, Twelve billion five hundred and seventy thousand two hundred and three.

3. Write in words, 4000005, 205000800, 16000600072, 870009080060, 312507618000.

4. Write in figures, Twenty-five million and twenty-five, Four billion sixteen million and fifteen.

5. Write in figures the smallest numbers of one, two, three, four, five, six and seven digits respectively.

6. Write in figures the largest numbers of one, two, three, four, five, six and seven digits respectively.

7. Read in words, 1111111, 101010101, 2002002002, 300030003, 40000400004, 1234567890, 5000006000006.

---

17. Roman Notation. The ancient Romans used the following letters or marks to denote numbers :

One.	Five.	Ten.	Fifty.	Hundred.	Five hundred.	Thousand.
I.	V.	X.	L.	C.	D.	M.

18. Principle. The number denoted by two or more of these letters together is the sum of their values taken singly, except when a lower letter precedes a higher, in which case the value of the lower letter is to be taken from that of the higher.

Thus II is 2, III is 3, VI is 6, but IV is 4, VII is 7, VIII is 8, XI is 11 but IX is 9, XV is 15, XVII is 17, XIX is 19, XX is 20, XL is 40, LX is 60, LXXXV is 85, XC is 90, CX is 110, CCLXXVI is 276, DCLXVI is 666, CDXLIV is 444.

EXERCISE IV.

Write in Arabic notation :

1. XVI, XXV, XXXIII, XLIX, XCIX, CLX, CCXXXII, CD, DC.

2. DCL, MDCLX, MMDCCXX, MCCCLXXXVII.

3. MDCCCLXXXII, MDLX, MDCL, MMDCLXVI, MCDXLIX.

19. To denote in Roman notation a number written in the Arabic notation, express the value of each digit, beginning with the highest, by one or more of the Roman letters, using as few as possible.

EXAMPLE 1. 14 (10 and 4) is XIV.

EXAMPLE 2. 269 (200, 60 and 9) is CCLXIX.

EXAMPLE 3. 444 is CDXLIV.

Note. A line over a letter increases its value a thousandfold. Thus $\overline{V}$ is 5,000, $\overline{X}$ is 10,000, $\overline{M}$ is 1,000,000.

EXERCISE V.

Express in Roman notation :

1. 48, 96, 162, 345, 547, 809, 900, 999.
2. 1008, 1234, 1357, 2468, 3060, 1012, 1500.
3. 4000, 5368, 11045, 27800, 630000.

◆

ADDITION.

20. When two or more numbers are collected into one number the operation is called **Addition.** The result is called the **Sum** of the numbers. The numbers which are to be added are called **Addends.**

Note. The unit of the several addends must be the same. Thus the sum of 5 pounds and 3 pounds is 8 pounds ; but we cannot add together 7 pounds and 4 yards.

21. Before learning the Rule for Simple Addition, it will be well for a child to learn the following Table, called the ADDITION TABLE. The child should satisfy himself that this Table is true by means of counters, or strokes on a slate.

ADDITION TABLE.

1 and	2 and	3 and	4 and	5 and
1 make 2	1 make 3	1 make 4	1 make 5	1 make 6
2 .. 3	2 .. 4	2 .. 5	2 .. 6	2 .. 7
3 .. 4	3 .. 5	3 .. 6	3 .. 7	3 .. 8
4 .. 5	4 .. 6	4 .. 7	4 .. 8	4 .. 9
5 .. 6	5 .. 7	5 .. 8	5 .. 9	5 .. 10
6 .. 7	6 .. 8	6 .. 9	6 .. 10	6 .. 11
7 .. 8	7 .. 9	7 .. 10	7 .. 11	7 .. 12
8 .. 9	8 .. 10	8 .. 11	8 .. 12	8 .. 13
9 .. 10	9 .. 11	9 .. 12	9 .. 13	9 .. 14
10 .. 11	10 .. 12	10 .. 13	10 .. 14	10 .. 15
6 and	7 and	8 and	9 and	10 and
1 make 7	1 make 8	1 make 9	1 make 10	1 make 11
2 .. 8	2 .. 9	2 .. 10	2 .. 11	2 .. 12
3 .. 9	3 .. 10	3 .. 11	3 .. 12	3 .. 13
4 .. 10	4 .. 11	4 .. 12	4 .. 13	4 .. 14
5 .. 11	5 .. 12	5 .. 13	5 .. 14	5 .. 15
6 .. 12	6 .. 13	6 .. 14	6 .. 15	6 .. 16
7 .. 13	7 .. 14	7 .. 15	7 .. 16	7 .. 17
8 .. 14	8 .. 15	8 .. 16	8 .. 17	8 .. 18
9 .. 15	9 .. 16	9 .. 17	9 .. 18	9 .. 19
10 .. 16	10 .. 17	10 .. 18	10 .. 19	10 .. 20

This Table can easily be carried on for numbers larger than 10 ; for instance, since 2 and 1 make 3, 2 and 11 make 10 more than 2 and 1, *i.e.* make 13. Again since 9 and 4 make 13, 9 and 14 will make 23, and so on, the result in each case being 10 more than in the corresponding case in the Table. Also 2 and 51 make 53, 9 and 54 make 63, and so on, the result in each case being 50 more than the corresponding result in the Table.

22. Sign of Addition. The sign $+$ (plus) means that the number following the sign is to be added to what goes before it.

Sign of Equality. The sign $=$ (equals, or is equal to) means that what goes before the sign is equal to what follows it

The sign $\therefore$ means "therefore."

The sign $\because$ means "because."

Brackets. When we wish to shew that the result of one operation is to be used in another operation, the numbers and signs of the former operation are put in brackets, ( ).

Example 1. To add to 6 the sum of 3 and 2 :

$$6 + (3 + 2) = 6 + 5 = 11.$$

Example 2. To prove that $3 + 2 = 2 + 3$:

$3 + 2 = (1 + 1 + 1) + (1 + 1) = 1 + 1 + 1 + 1 + 1,$
$2 + 3 = (1 + 1) + (1 + 1 + 1) = 1 + 1 + 1 + 1 + 1,$
$\therefore 3 + 2 = 2 + 3.$

Hence addends may be taken in any order.

Example 3. Find the sum of 5, 3 and 7.

$$5 + 3 + 7 = 15,$$

or

$$\begin{array}{r} 7 \\ 3 \\ 5 \\ \hline 15 \end{array}$$

Explanation. 5 and 3 make 8. 8 and 7 make 15.

Note. Use as few words as possible. Thus instead of saying 5 and 3 make 8, 8 and 7 make 15, say 5, 8, 15.

Example 4. Find the sum of 3, 5, 7 and 9.

$3 + 5 + 7 + 9 = 24.$ *Explanation.*

$$\text{or}\quad\begin{array}{r}3\\5\\7\\9\\\hline 24\end{array}\qquad\begin{array}{l}\text{say } 3, 8, 15, 24,\\ \\ \\ \text{or } 9, 16, 21, 24.\\ \\ \end{array}$$

EXERCISE VI.

1. Find the sum of 3, 2, 1, 6 and 8.

2. Find the result of $5 + 2 + 3 + 4 + 6$.

3. Add together 8, 4, 3, 5, 6 and 7.

4. A boy received 3 merit marks on Monday, 2 on Tuesday, 4 on Wednesday, 5 on Thursday, and 1 on Friday. How many did he get during the week?

5. A boy spent 3 cents on apples, 5 cents on candies, 4 cents on a pencil, and had 8 cents left. How many cents had he at first?

6. Find the sum of the first six integers.

---

23. To add larger numbers.

Example 1. Find the sum of 15, 18, 13.

$$15 + 18 + 13 = 46,$$

$$\text{or}\quad\begin{array}{r}15\\18\\13\\\hline 46\end{array}$$

Explanation. First add the units, saying, 3, 11, 16 : put down 6 and add 1 to the tens. Then add the tens, saying, 1, 2, 3, 4 : set down 4 under the column of tens.

EXAMPLE 2. Add 27, 35, 18.

27
35
18
—
80

Explanation. Add the units, saying, 8, 13, 20: 0 and carry 2 tens. Then add the tens, saying, 2, 3, 6, 8.

EXAMPLE 3. Add together 47, 385, 7 and 236.

47
385
7
236
——
675

Explanation. Add the units, saying, 6, 13, 18, 25: 5 and carry 2. Then add the tens, saying, 2, 5, 13, 17: 7 and carry 1. Finally, for the hundreds say, 1, 3, 6.

EXAMPLE 4. Find the sum of twenty-six, seven hundred and eight, eighty-five, four thousand and sixty-three, ninety-four, six hundred and nineteen.

26
708
85
4063
94
619
——
5595

Explanation. 9, 13, 16, 21, 29, 35: 5 and carry 3. 3, 4, 13, 19, 27, 29: 9 and carry 2. 2, 8, 15: 5 and carry 1. 1, 5.

EXERCISE VII.

1.	2.	3.	4.
5	9	8	3
3	5	7	5
8	7	5	7
9	6	4	9

5.	6.	7.	8.
12	17	10	14
13	11	15	8
14	19	18	17
15	16	9	5

9.	25	10.	34	11.	57	12.	65
	18		19		36		87
	27		28		28		79
	14		17		49		98
13.	176	14.	275	15.	457	16.	785
	158		87		700		47
	162		140		876		364
	147		8		593		8
17.	1307	18.	3185	19.	516	20.	8369
	968		4506		6879		748
	1075		578		3458		9587
	836		6954		98		5400
	57		3065		506		83
21.	638	22.	9	23.	987	24.	8
	47		19		876		80
	5806		199		765		808
	7319		1999		654		8008
	876		9000		543		888
	55		900		432		88

25. Find the result of 7 + 5 + 9 + 6 + 4; of 15 + 17 + 12 + 19; and of 26 + 37 + 48 + 59 + 70.

26. There are in a church 37 men, 45 women, 28 boys, and 26 girls. How many are there altogether?

27. A farmer sewed 36 acres with wheat, 19 acres with barley, 17 acres with oats, 8 acres with Indian corn. He has besides 28 acres in fallow, 35 acres of meadow, 12 acres of bush, and the garden, orchard and buildings take up 15 acres. How much land has he?

28. In the first car of a train there are forty-six persons; in the second, fifty-three; in the third, thirty-eight; in the fourth, twenty-nine. Counting in the driver, stoker, conductor, newsboy, and two brakesmen, how many persons are there on the train?

29. Find the sum of sixty, six hundred and six, six thousand six hundred, six thousand and sixty, sixty thousand and sixty, and sixteen.

30. Find the sum of all the numbers of three digits that can be made with the figures 5, 7 and 9.

31. In a game of cricket, the first man out made 15 runs, the second 9, the third 28, the fourth and fifth none, the sixth 17, the seventh and eighth 7 each, the ninth 5, the tenth 3, and the last man carried out his bat for 6. Also there were 13 byes and 8 wides. Find the total score.

32. On a certain section there are four families, comprised respectively of nine, eleven, eight and seven persons. How many people live on the section?

33. John had 5 hens, which laid respectively 15, 19, 18 and 17 eggs. How many eggs had he in all?

34. A man bought a pound of cheese for 17 cents, a pound of butter for 21 cents, two pounds of sugar at 9 cents a pound, and a box of matches for 18 cents. How much did he spend?

35. Find the result of $67 + 38 + 29 + 56 + 135$.

36. One man walks 35 miles each day. Another walks 41 miles the first day, 38 the second day, and so on, each day 3 miles less than the day before. Shew that in 5 days they will have both walked the same distance.

General Rule for Addition.

24. Write down the given numbers under each other, so that units may come under units, tens under tens, hundreds under hundreds, and so on: then draw a line under the lowest number.

Find the sum of the column of units: if it be less than ten, write it down under the column of units below the line just drawn, but if it be greater than ten, then write down the units' figure (*i. e.* the last figure on the right hand) of the sum under the column of units, and carry to the column of tens the remaining figure or figures.

Add the column of tens and the figure or figures you carry as you have added the column of units, and treat its sum in exactly the same way as you have treated the column of units.

Treat each succeeding column (viz. hundreds, thousands, &c.) in the same way.

Write down the full sum of the last column on the left hand.

The entire sum thus obtained will be the sum or amount of the given numbers.

25. TEST. The test of correctness is that the result when you add downwards should be the same as when you add upwards.

Note. It is useful to set down the numbers carried under the column to which they are carried, so that when the result of any column added downwards does not agree with that got by adding upwards, we may not have to add the previous columns over again.

EXAMPLE :

4785
964
607
5385
8046
97
386
6758

27028
454

Explanation. 8, 14, 21, 27, 32, 39, 43, 48: 8 units, and set down 4 under the tens' column. Then 4, 9, 17, 26, 30, 38, 44, 52: 2 tens, and set down 5 under the column of hundreds. Then 5, 12, 15, 18, 24, 33, 40: 0 hundreds, and set down 4 under the column of thousands. Lastly, 4, 10, 18, 23, 27: set down 27.

EXERCISE VIII.

1. 86
359
48
9
5763
865

2. 549
67
4396
5804
937
55

3. 938
547
689
456
878
936

4. 8756
905
87
519
9860
56

5. 8100
6486
785
39
5008
654

6. 649
5876
87
595
9672
8469

7. 98
87
76
65
54
43

8. 77
777
7777
7777
777
77

9. 789
798
897
879
987
978

10. 365
806
97
658
9
476
59

11. 667
5984
875
79
6586
953
78

12. 6158
9476
8795
7346
9518
5678
4937

13. Find the sum of six hundred and nine, five thousand and sixty-eight, seventy-six, forty-seven thousand and nineteen, fifty thousand eight hundred, six thousand four hundred and twelve, and eighty-five.

14. Find the result of 28 + 136 + 79 + 8 + 347 + 19 + 508 + 600.

15. A battalion was composed of eight companies, having respectively 65, 58, 47, 63, 56, 48, 60 and 55 men. The band numb red 28, and there were 19 officers. Find the total number in the battalion.

16. An oil well was bored in 15 days. 50 feet was bored the first day, and each day afterwards 3 feet less than the day before. Find the total depth.

17. A family spends 152 dollars on clothing, 144 dollars on rent, 416 dollars on provisions, 65 dollars on small expenses, and put 36 dollars in the Savings' Bank. Find the income of the family.

18. A farmer sold a cow for 45 dollars, two bullocks for 58 dollars each, and a horse for as much as the other three animals put together. How much money did he receive?

19. In a school there are forty-eight in the first class, thirty-seven in the second class, twenty-six in the third class, nineteen in the fourth class, and seventeen in the fifth class. Find the whole number in the school.

20. A man bought 4 horses. For the first he paid 97 dollars; for the second 18 dollars more than for the first; for the third 15 dollars more than for the second; and for the fourth as much as for the other three. Find how much he spent altogether.

21. A Boer doctor being asked his fee for visiting a patient 20 miles off, said, "a dollar a mile." At the end of the first mile he said, "one mile, one dollar," and got his dollar. At the end of the second mile he said, "two miles, two dollars." Upon this his conductor turned back. What would have been the doctor's fee?

22. Find the sum of the first twelve odd numbers.

23. A gentleman left three thousand five hundred dollars to each of his three sons, to each of his two daughters seven hundred and fifty dollars more than a son's share, and to his wife one thousand dollars more than the shares

of a son and a daughter put together. Find the amount of his property.

24. The exports of Canada for November, 1881, were, in value, minerals, 133,809 dollars ; fish, 723,141 dollars ; timber, 1,975,978 dollars ; animals, 2,172,295 dollars ; agricultural products, 6,694,710 dollars ; manufacture, 396,415 dollars ; and miscellaneous articles, 43,088 dollars. Find the total exports for November.

◆

SUBTRACTION.

26. When a less number is taken from a greater number, the operation is called Subtraction.

The former number is called the **Subtrahend**, and the latter the **Minuend.**

The result is called the **Remainder**, or **Difference.**

27. Sign of Subtraction. The sign – (minus) means that the number which follows the sign is to be taken from that which goes before it. Thus: $2 - 1 = 1$. $3 - 1 = 2$.

Note 1. We have seen that $3 + 2 = 2 + 3$, but it is not true that $3 - 2$ is equal to $2 - 3$, for $3 - 2 = 1$, but $2 - 3$ means that 3 is to be taken from 2, which cannot be done.

Note 2. To subtract 5 from 8 is the same as to find what number added to 5 makes 8. Hence Subtraction is called the converse of Addition.

Note 3. The Minuend and Subtrahend must have the same unit, which will also be the unit of the result. Thus, if from 5 pencils we take 3 pencils, the remainder is 2 pencils ; but we cannot take 3 cents from 5 pencils.

SUBTRACTION TABLE.

1 from	2 from	3 from	4 from	5 from
1 leaves 0	2 leaves 0	3 leaves 0	4 leaves 0	5 leaves 0
2 .. 1	3 .. 1	4 .. 1	5 .. 1	6 .. 1
3 .. 2	4 .. 2	5 .. 2	6 .. 2	7 .. 2
4 .. 3	5 .. 3	6 .. 3	7 .. 3	8 .. 3
5 .. 4	6 .. 4	7 .. 4	8 .. 4	9 .. 4
6 .. 5	7 .. 5	8 .. 5	9 .. 5	10 .. 5
7 .. 6	8 .. 6	9 .. 6	10 .. 6	11 .. 6
8 .. 7	9 .. 7	10 .. 7	11 .. 7	12 .. 7
9 .. 8	10 .. 8	11 .. 8	12 .. 8	13 .. 8
10 .. 9	11 .. 9	12 .. 9	13 .. 9	14 .. 9

6 from	7 from	8 from	9 from	10 from
6 leaves 0	7 leaves 0	8 leaves 0	9 leaves 0	10 leaves 0
7 .. 1	8 .. 1	9 .. 1	10 .. 1	11 .. 1
8 .. 2	9 .. 2	10 .. 2	11 .. 2	12 .. 2
9 .. 3	10 .. 3	11 .. 3	12 .. 3	13 .. 3
10 .. 4	11 .. 4	12 .. 4	13 .. 4	14 .. 4
11 .. 5	12 .. 5	13 .. 5	14 .. 5	15 .. 5
12 .. 6	13 .. 6	14 .. 6	15 .. 6	16 .. 6
13 .. 7	14 .. 7	15 .. 7	16 .. 7	17 .. 7
14 .. 8	15 .. 8	16 .. 8	17 .. 8	18 .. 8
15 .. 9	16 .. 9	17 .. 9	18 .. 9	19 .. 9

Note. The use of this table can be extended. Thus : 9 from 14 leaves 5. 9 from 24 leaves 15. 8 from 12 leaves 4. 8 from 32 leaves 24.

As few words as possible should be used. Thus : 9 from 14, 5. 9 from 24, 15.

EXERCISE IX.

1. A boy spent 3 cents out of 10 cents. How many had he left?

2. Out of a class of 12 pupils 5 are absent. How many are present?

3. A fishing pole, 17 feet long, had 8 feet broken off. How long was the remainder?

4. A boy has 9 cents. How many more must he get to have 15?

5. Out of 16 words, a pupil spelt 9 correctly. How many were missed?

6. A boy who had 25 cents bought two oranges for 6 cents, and a ball for five cents. How much had he left?

7. Give the results for $7 - 2$, $9 - 4$, $17 - 6$, $15 - (6 + 2)$, $(6 + 4) - (3 + 2)$.

---

28. To subtract, when the numbers are greater than 10.

Example 1. To subtract 13 from 17.

17
13
—
4

Explanation. 3 units taken from 7 units leaves 4 units. 1 ten from 1 ten leaves nothing.

Example 2. From 35 take 18.

35
18
—
17

Explanation. 8 units cannot be taken from 5 units, but we can take 1 of the 3 tens along with the 5 units and say 8 from 15 leaves 7. Then 1 ten from the remaining 2 tens leaves 1 ten.

Or add 10 units to the 5 units and say 8 from 15, 7, and make up by taking 2 tens instead of 1 ten from the 3 tens, which leaves 1 ten.

Example 3. Find the difference between 218 and 176.

218 *Explanation.* 6 from 8, 2. 7 from 11, 4. 2
176 from 2, nothing.

42

EXERCISE X.

1. From 18 Take 6	2. 25 15	3. 39 28	4. 77 55
5. 96 72	6. 89 48	7. 16 9	8. 17 8
9. 172 83	10. 568 486	11. 385 99	12. 444 289
13. 878 789	14. 839 747	15. 29 19	16. 380 299
17. 564 489	18. 901 784	19. 1000 999	20. 583 347
21. 610 589	22. 309 289	23. 712 619	24. 782 695
25. 867 789	26. 908 888	27. 645 586	28. 912 868
29. 546 457	30. 128 89	31. 265 176	32. 800 753

33. Subtract twenty-five from forty-nine; seventy-three from ninety-nine; eighty-eight from one hundred and one; fifty-six from eighty-one.

34. Find the difference between one hundred and eleven and seventy-nine; two hundred and five and one hundred and sixty-two.

35. On a farm there are in one field thirty-four sheep, in another field there are forty-four. Find the number

of sheep in the two fields, and also how many more sheep there are in the one field than in the other.

36. Thomas had 208 beech-nuts, he gave William 69, Andrew 103. How many had he left?

37. Find the sum of twenty-two and thirty-one, also their difference.

38. A boy was given by his father 25 cents to buy a newspaper for 3 cents, 3 slate pencils for 2 cents, 2 postage stamps for 3 cents each. What change should he bring home?

General Rule for Subtraction.

29. Write down the less number under the greater number, so that units may come under units, tens under tens, hundreds under hundreds, and so on; then draw a straight line under the lower number.

Take, if you can, each figure of the lower number from the figure of the upper number which stands directly over it, and place the remainder under the line just drawn, units under units, tens under tens, and so on.

But, if any figure in the lower number be greater than the figure just above it, then add ten to the upper figure, and then subtract the lower figure from the upper figure thus increased, and write down the remainder as before.

Add one to the next figure in the lower number, and then take this figure thus increased from the figure just above it, by one of the methods already explained.

Go on thus with all the figures.

The result thus obtained will be the difference or remainder.

30. Test. The test of the correctness of the result is that the remainder added to the subtrahend should produce the minuend.

Example. How much greater is 8312 than 5482 ?

8317 *Explanation.* 2 from 7, 5. 8 from 11, 3.
5482 5 from 13, 8. 6 from 8, 2.

2835

Test. 8317

EXERCISE XI.

1. From 2568
Take 1456

2. 3784
2642

3. 3965
2747

4. 67850
38467

5. 80635
72747

6. 63201
28363

7. 364061
276593

8. 235063
178958

9. 586735
398868

10. 28685
19756

11. 30802
29684

12. 31507
28789

13. 600501
398506

14. 400506
327807

15. 816655
445738

16. 863051
784584

17. 206080
327807

18. 368531
269278

19. 30060000
21162222

20. 3561111
2782233

21. 600000
110131

22. 908070
129389

23. 235678
123989

24. 4689
3689

25. What number taken from two thousand and five will leave 207 ? What number added to seven hundred thousand and forty-five will make one million five thousand and fifty-three ?

26. Find the sum of 79206, 80912, 30019 ; and from the sum take the difference between eighty-five thousand and twelve, and fifty-six thousand two hundred and four.

27. The sum of three numbers is twenty thousand and three hundred ; the first is 5062, and the second is fifteen

hundred and fourteen more than the first. Find the third number.

28. When will the Prince of Wales, who was born in the year 1841, be as old as the Queen now (1882) is, who was born in the year 1819? How old will the Queen then be?

29. What is the difference between 20001 + 2010 − 1915 + 747 − 809 and 1500 − 782 + 1189 − 387?

30. The Indian Mutiny took place in 1857; how many years was that before 1882?

31. A wheat buyer bought 5,916, 8,014, and 11,129 bushels of wheat, and sold 9,697 bushels; how many bushels had he left in stock?

32. Out of an army of forty thousand men, 30,012 are ordered on foreign service: how many remain at home?

33. If in a school of 368 scholars, 93 are admitted and 79 leave during the year, what is the number at the end of the year?

34. A person writing in 1860 says the battle of Waterloo took place 45 years ago: find the date of the battle.

35. A man whose yearly income is 1,200 dollars, spends 100 dollars for house rent, 72 dollars for servants, 25 dollars in travelling, 120 dollars in clothing, 825 dollars in household bills; will he have saved anything or be in debt at the end of the year, and to what amount?

36. The population of the Counties on the river St. Lawrence in 1881, was 119,773; that of those on the Ottawa river was 148,775; find the difierence between the population of these Counties.

37. Find the result of 801 + 317 — 295 + 300 — 283 + 675 — 863.

38. For the year 1880, the exports of Canada were 87,911,458 dollars, and the imports 86,489,747 dollars; find by how much the exports exceeded the imports.

39. Find the difference between three million five thousand and ten, and two million twenty thousand and eight,

40. The minuend is five million eight thousand and twelve, the subtrahend four million six hundred and twenty-three thousand; find the remainder.

41. What number must be added to the sum of 803, 61133 and 5069 to make the result equal to the sum of 60512, 893, 27 and 90438?

42. Find the final remainder after subtracting 603956 as many times as possible in succession from 2875612.

◆

MULTIPLICATION.

31. When a number is taken as an addend any number of times, the operation is called **Multiplication.**

The number which is taken as an addend is called the **Multiplicand.**

The number of times it is taken is called the **Multiplier.**

The result is called the **Product.**

32. **Sign of Multiplication.** The sign ×, (into, or multiplied by), between two numbers means that one number is to be multiplied by the other.

EXAMPLE 1. 2×3 means 2 multiplied by 3 (2 taken 3 times), or twice 3 (3 taken twice).

Note 1. We get the same result whether we make 2 or 3 the multiplier, for $3 + 3 = 6$, and $2 + 2 + 2 = 6$.

Note 2. The multiplier, or number of times the multiplicand is to be taken, must always be abstract, that is, a mere number.

The unit of the product is the same as that of the multiplicand. Thus, 5 dollars taken three times is 15 dollars; 4 yards taken twice is 8 yards.

33. **Multiplication Table.** All products, from 1 taken once, to 12 taken 12 times, are put into a Table which

must be learned so well that the pupil may be able to give any product promptly.

MULTIPLICATION TABLE.

Twice	3 times	4 times	5 times	6 times	7 times
1 makes 2	1 makes 3	1 makes 4	1 makes 5	1 makes 6	1 makes 7
2 .. 4	2 .. 6	2 .. 8	2 .. 10	2 .. 12	2 .. 14
3 .. 6	3 .. 9	3 .. 12	3 .. 15	3 .. 18	3 .. 21
4 .. 8	4 .. 12	4 . 16	4 .. 20	4 .. 24	4 .. 28
5 .. 10	5 .. 15	5 .. 20	5 .. 25	5 .. 30	5 .. 35
6 .. 12	6 .. 18	6 .. 24	6 .. 30	6 .. 36	6 .. 42
7 .. 14	7 .. 21	7 .. 28	7 .. 35	7 .. 42	7 .. 49
8 .. 16	8 .. 24	8 .. 32	8 .. 40	8 .. 48	8 .. 56
9 .. 18	9 .. 27	9 .. 36	9 .. 45	9 .. 54	9 .. 63
10 .. 20	10 .. 30	10 .. 40	10 .. 50	10 .. 60	10 .. 70
11 .. 22	11 .. 33	11 .. 44	11 .. 55	11 .. 66	11 .. 77
12 . 24	12 .. 36	12 . 48	12 .. 60	12 .. 72	12 .. 84

8 times	9 times	10 times	11 times	12 times
1 makes 8	1 makes 9	1 makes 10	1 makes 11	1 makes 12
2 .. 16	2 .. 18	2 .. 20	2 .. 22	2 . 24
3 .. 24	3 .. 27	3 .. 30	3 .. 33	3 .. 36
4 .. 32	4 .. 36	4 .. 40	4 .. 44	4 .. 48
5 .. 40	5 .. 45	5 .. 50	5 .. 55	5 .. 60
6 .. 48	6 .. 54	6 .. 60	6 .. 66	6 .. 72
7 .. 56	7 .. 63	7 .. 70	7 .. 77	7 .. 84
8 .. 64	8 .. 72	8 .. 80	8 .. 88	8 .. 96
9 .. 72	9 .. 81	9 .. 90	9 .. 99	9 .. 108
10 .. 80	10 .. 90	10 .. 100	10 .. 110	10 .. 120
11 .. 88	11 .. 99	11 . . 110	12 .. 121	11 .. 132
12 .. 96	12 .. 108	12 .. 120	12 .. 132	12 .. 144

Note. Questions and answers on the Table should be given as briefly as possible. Thus : *Question.* Seven times five ? *Answer.* Thirty-five.

EXERCISE XII.

1. If one orange costs 5 cents, how many cents will 7 cost ?

2. If a boy reads 6 pages a day, how many will he read in 5 days ?

3. A boy walks a distance of two miles and back every day. How many miles does he walk in a week ?

4. How many months has a boy lived who is nine years old ?

5. A boy is sent with 75 cents to buy 8 pounds of sugar at 9 cents a pound. How many cents will he have left ?

6. A farmer exchanges 8 dozen eggs, at 12 cents a dozen, for 11 yards of calico at 9 cents a yard. How many cents will he have to pay ?

7. Find the results of $(5 + 3) \times (7 + 3)$. $(6 + 5) \times (9 - 4)$. $(8 - 2) \times (12 - 3)$. $7 \times 5 + 8$. $8 \times 7 - 9 \times 6$. $11 + 6 \times 8$. $100 - 8 \times 9$.

---

34. To multiply when the multiplicand is greater than 12, but the multiplier not greater than 12.

EXAMPLE 1. Multiply 17 by 3.

$$\begin{array}{r}17\\3\\\hline 51\end{array}$$

Explanation. 3 times 7 units makes 21 units ; set down 1 unit and add (carry) 2 to the tens. Then 3 times 1 ten makes 3 tens, or, altogether, 5 tens.

Example 2. Multiply 287 by 5.

287 *Explanation.* 5 times 7 units, 35 units; 5 and
5 carry 3 tens. 5 times 8 tens, 40 tens, which, with
—— 3, make 43 tens; 3 tens and carry 4 hundreds. 5
1435 times 2 hundreds, 10 hundreds, which, with 4, make 14 hundreds.

Example 3. If a boy sleep nine hours every night, how many hours will he sleep in a year (365 days)?

365 *Explanation.* 9 times 5, 45; 5 and carry 4.
9 9 times 6, 54, and 4, 58; 8 and carry 5. 9 times
—— 3, 27, and 5, 32.
3285 ∴ The result is 3285 hours.

Note. The operation is performed with the numbers merely. The unit of the result is known from the question.

In this example 9 hours is to be taken as an addend 365 times, therefore the unit of the result is an hour.

Since 9 taken 365 times makes the same number as 365 taken 9 times, we perform the latter operation which is the shorter one.

EXERCISE XIII.

Multiply		1.	53	2.	47	3.	88	4.	48	5.	60
	By		2		2		2		3		3
6.	29	7.	27	8.	83	9.	90	10.	67	11.	36
	3		4		4		5		5		6
12.	99	13.	27	14.	53	15.	45	16.	77	17.	69
	6		7		7		8		8		9
18.	54	19.	20	20.	99	21.	53	22.	87	23.	60
	9		10		10		11		11		12
24.	49	25.	687	26.	800	27.	276	28.	777	29.	497
	12		2		3		4		5		6
30.	905	31.	835	32.	487	33.	560	34.	538	35.	704
	7		8		9		10		11		12

36. Supposing an acre of land to produce 25 bushels of wheat, how many bushels will 9 such acres produce, and what will be their value at 130 cents a bushel?

37. A boy takes 75 steps in a minute; how many steps can be taken in 11 minutes?

38. If in each of 8 barrels of water there are 36 gallons, how many gallons in all?

39. A book has 375 pages; how many pages in 11 such books?

---

35. Rule for Multiplication, when the multiplier is a number not larger than 12.

Place the multiplier under the multiplicand, units under units, and (if the multiplier be 10, 11, or 12), ten under tens; then draw a line under the multiplier.

Multiply each figure of the multiplicand, beginning with the units, by the figure or figures of the multiplier (by means of the Multiplication Table).

Write down and carry as in Simple Addition.

Example. If a car carries 375 bushels of wheat, how many bushels in 12 car-loads?

$$\begin{array}{r} 375 \\ 12 \\ \hline 4500 \end{array}$$

Explanation. 12 times 5, 60 : 0 and carry 6. 12 times 7, 84, and 6, 90 : 0 and carry 9. 12 times 3, 36, and 9, 45.

EXERCISE XIV.

Multiply	1. 5849	2. 9873	3. 6057	4. 97068
By	2	3	4	5
5. 69360	6. 80965	7. 439090	8. 48508	9. 38476
6	5	7	8	9
10. 49216	11. 69432	12. 21357	13. 91537	
12	12	11	12	

14. Multiply 65030807 by each of the following, 2, 5, 3, 7, 4, 9, 6, 8, 11 and 12.

15. Two men start from the same place and travel in opposite directions, one at the rate of 40 miles a day, the other at the rate of 35 miles a day ; how far apart will they be at the end of six days ?

16. In question 15, if the men travelled in the same direction, how far apart would they be at the end of eight days ?

17. If the second man at the end of two days turn back, and travel each day in the opposite direction the same number of miles as before ; find how far apart they will be at the end of the six days.

18. How far will a train travelling 45 miles an hour run in 8 hours, if an hour is consumed in stoppages ?

19. A farmer sold 5 cords of wood for 4 dollars a cord, 7 sheep at 6 dollars each, and 2 cows for 30 dollars each ; he bought a chain worth 3 dollars, a plough worth 18 dollars, 20 pounds of sugar at 10 cents a pound, 6 pounds of tea at 50 cents a pound, a pair of pants for 4 dollars ; find the sum of money he should take home, supposing that his expenses were a dollar.

20. There were 211 school days in 1881. A boy who lives a mile from school was absent 10 days in the year. If he comes home to dinner at noon every day he is at school, find the number of miles he walks to and from school during the year.

21. If the area of the Province of Ontario be 180,000 square miles, and the average population be 11 persons to the square mile, find the total population.

36. Continued Multiplication. When one number is multiplied by another, and the product multiplied by a third number, and so on, the operation is called continued multiplication, and the result is called the continued product of the numbers.

Factors. The numbers whose continued product makes a certain number are called the factors of that number.

Note. Although 1 is a factor of all numbers, it is not usually counted in the number of factors.

Thus the factors of 6 are 1, 2 and 3, or $1 \times 2 \times 3 = 6$, but we commonly say that the factors of 6 are 2 and 3.

Composite Numbers are those which are the products of two or more factors.

Prime Numbers are those which are not the products of two or more other numbers.

Thus, since 6 is 3 times 2, and 10 is 5 times 2, 6 and 10 are composite numbers, but 1, 3, 5, 7, 11 and so on are prime numbers.

---

37. To multiply by composite numbers whose factors are not greater than 12.

EXAMPLE 1. Multiply 137 by 14.

$$\begin{array}{r} 137 \\ 7 \\ \hline 959 \\ 2 \\ \hline 1918 \end{array}$$

Explanation. We first find 7 times 137 by multiplying by 7, and then take this result twice by multiplying it by 2.

EXAMPLE 2. Multiply 564 by 126.

$$\begin{array}{r} 564 \\ 9 \\ \hline 5076 \\ 7 \\ \hline 35532 \\ 2 \\ \hline 71064 \end{array}$$

Explanation. We can easily see that $126 = 63 \times 2$ and $63 = 9 \times 7$, so that $126 = 9 \times 7 \times 2$. We first take 9 times 564. Then take this result 7 times, which gives 63 times 564. Finally we take this result twice, which gives 126 times 564.

38. The most important use of continued multiplication is when the multiplier ends with one or more 0's.

EXAMPLE 1. Multiply 638 by 80.

$$\begin{array}{r} 638 \\ 8 \\ \hline 5104 \\ 10 \\ \hline 51040 \end{array}$$

or more briefly

$$\begin{array}{r} 638 \\ 80 \\ \hline 51040 \end{array}$$

Explanation. In the shorter form we multiply by 8 and affix a 0, which, by moving the other figures one place to the left, increases the value tenfold.

EXAMPLE 2. Multiply 743 by 600.

$$\begin{array}{r} 743 \\ 600 \\ \hline 445800 \end{array}$$

Explanation. We multiply 643 by 6 and affix two 0's to the product, which multiplies it by 100.

EXAMPLE 3. Multiply 527 by 28000.

$$\begin{array}{r} 527 \\ 7000 \\ \hline 3689000 \\ 4 \\ \hline 14756000 \end{array}$$

Explanation. 28000 = 1000 × 7 × 4. We multiply 527 by 7 and affix 3 ciphers to multiply by 1000, and then multiply the result by 4.

EXERCISE XV.

Multiply

1. 517 by 45
2. 3684 by 60
3. 87 by 440
4. 927 by 800
5. 46 by 105
6. 174 by 88
7. 843 by 1200
8. 92 by 630
9. 365 by 3200
10. 283 by 324
11. 570 by 350
12. 800 by 900
13. 17000 by 360
14. 47 by 56
15. 315 by 75
16. 837 by 216
17. 58 by 49
18. 69 by 64
19. 345 by 81
20. 76 by 128

21. Find the continued product of 1, 2, 3, 4, 5, 6.
22. Find the value of 2 × 2 × 2 × 2 × 2 × 2.
23. Find the result of 3 × 3 × 3 × 3 × 3.
24. Find the result of 4 × 4 × 4 × 4.
25. Find the result of 5 × 5 × 5.

---

39. To multiply by any number greater than 12.

Example 1. Multiply 946 by 17.

946	or	946
17		17
6622		6622
9460		946
16082		16082

Explanation. We find 7 times 946 by multiplying by 7, and then 10 times 946 by affixing a cipher to 946. The two products added together give 17 times 946. We may omit the 0 if we take care to move the 946 one place to the left.

Example 2. Multiply 1358 by 206.

1358	or	1358
206		206
8148		8148
271600		2716
279748		279748

Explanation. We first multiply by 6, then by 200. The latter part is done by multiplying 1358 by 2 and affixing two 0's. The 0's affixed may be omitted if we take care to begin the lower line two places to the left of the unit figure. The sum of the two products gives us 206 times 1358.

Example 3. Multiply 3570 by 4300.

3570
4300
1071000
14280
15351000

Explanation. We first take the multiplicand 300 times by multiplying it by 3 and affixing two 0's. Then we take it 4000 times by multiplying it by 4, placing the first figure in the fourth place as there are 3 ciphers to be affixed, although they are omitted to save work. The sum of the two products is 4300 times 3570.

Example 4. Multiply 34718 by 200706.

$$\begin{array}{r} 34718 \\ 200706 \\ \hline 208308 \\ 243026 \\ 69436 \\ \hline 6968110908 \end{array}$$

Explanation. We multiply 34718 by 6, by 700 and by 200000, omitting the 0's to be affixed, but beginning the second product in the third place, and the third product in the sixth place.

Note. It may be seen from the above examples that when the affixed ciphers are omitted, the first figure of each product begins under the multiplier for that product.

EXERCISE XVI.

Multiply	1. 275	2. 382	3. 708	4. 674
By	13	23	29	31

5. 718	6. 789	7. 986	8. 893
37	53	65	61

9. 3832	10. 5832	11. 80695
73	92	79

12. 70683 by 521
13. 86075 by 483
14. 57806 by 600
15. 98760 by 473
16. 46802 by 632
17. 33680 by 464
18. 86456 by 336
19. 40632 by 508
20. 97008 by 413
21. 263400 by 370
22. 396005 by 250
23. 440062 by 20860
24. 550032 by 23022
25. 386033 by 64006
26. 454005 by 3806
27. 1101010 by 83085
28. 332266 by 460600
29. 456300 by 243040
30. 226600 by 35624

31. Required the product of two million sixty thousand and forty-nine, and sixty-eight thousand seven hundred and thirty-five.

32. Multiply (1) 82679 by 180, and by its factors 4, 5, and 9; and (2) 75893 by 1188, and by its factors 9, 11 and 12.

33. A person buys 12873 acres of land in Manitoba at 13 dollars an acre: how much did he have to pay for the whole?

34. A drover bought 617 sheep at 5 dollars each, 215 head of cattle at 25 dollars each, and 812 lambs at 4 dollars each; how much did he have to pay altogether?

35. A Jewish shekel weighed 219 grains troy; what was the weight (in grains) of a talent, containing 30000 shekels?

36. Find the value of the free-will offering of 16750 shekels made by the Captains of Israel, after the destruction of Midian, the Jewish shekel being worth 56 cents.

37. What is the price of 1287 bushels of oats at 36 cents a bushel?

38. In fencing, 6 rails are required for every 10 feet; find the number of rails required for fencing a ten acre field, the distance round being 2640 feet.

39. In walking, a boy steps two feet each time, and takes 100 steps a minute: what distance will he walk in an hour or 60 minutes?

40. The distance from Toronto to Collingwood by the N. R. is 93 miles: find the cost of making this road at 22500 dollars a mile.

40. General Rule for Multiplication, when the Multiplier is any number larger than 12.

Place the multiplier under the multiplicand, units under units, tens under tens, and so on; then draw a line under the multiplier.

Multiply each figure of the multiplicand, beginning with the units, by the figure in the units' place of the multiplier (by means of the table given for Multiplication); write down and carry as in Addition.

Then multiply each figure of the multiplicand, beginning with the units, by the figure in the tens' place of the multiplier, placing the first figure so obtained under the tens of the line above, the next figure under the hundreds, and so on.

Proceed in the same way with each succeeding figure of the multiplier.

Then add up all the results thus obtained by the rule of Simple Addition.

Example. Multiply 1758 by 367.

```
  1758
   367
 ------
 12306
10548
5274
------
645186
```

Explanation. We first multiply 1758 by 7, then by 6, placing the first figure one figure to the left on account of the omitted 0. Then multiply by 3, placing the first figure two places to the left on account of the two 0's omitted.

Test of Correctness. If the multiplier be made the multiplicand, and the multiplicand the multiplier, the result should be the same as before.

EXERCISE XVII.

```
1.  3856          2.  14089          3.  7654
      73                 87                97

4.  4387          5.  92068          6.  38269
     860                374               704

7.  5136          8.  6478           9.  43785
     879                906               379
```

10. 13587 1708	11. 708056 2078	12. 940068 50093
13. 360785 40907	14. 6009853 5008	15. 716549 68758
16. 800600 70086	17. 53400 8750	18. 78000 57009
19. 4768 3967	20. 154716 8165	21. 3790 58600
22. 38965 54070	23. 81679 500800	24. 37948 5679

Multiply

25. 37219678 by 907006

26. 8219543 by 700761

27. 7051136 by 803106

28. 6644585 by 437980

29. 7040957 by 900700

30. 6789123 by 721369

31. 5947687 by 056072

32. 6784365 by 446709

33. 8097563 by 580039

34. 9927859 by 796030

35. Find the product of three thousand five hundred and eighteen by two thousand four hundred and fourteen ; also of ten thousand and sixty-six by five thousand six hundred and twenty-eight.

36. If an acre of land contains 4840 square yards, how many square yards are there in 23 acres ?

37. How many matches are there in 568 boxes, each containing 125 ?

38. A book has 350 pages, 35 lines on a page and 53 letters in a line ; how many letters in the whole book ?

39. Sound travels at the rate of 1120 feet a second; how far will it travel in an hour (3600 seconds)?

40. Find the continued product of 23, 26, 417 and 9673.

41. Find the product of five thousand and twenty-five and three thousand and thirty-three; of eight thousand and seventeen and four thousand and sixteen.

42. By how much does the product of the sum and difference of a thousand two hundred and five and fifteen thousand three hundred and eight exceed the product of a thousand and twelve hundred and three.

◆

DIVISION.

41. When we find how many times one number can be subtracted from another, or, in other words, how many times it is contained in another, the operation is called **Division.**

Divisor. The first number is called the **Divisor.**

Dividend. The second number is called the **Dividend.**

Quotient. The result, or number of times, is called the **Quotient.**

Remainder. If after the divisor has been subtracted as often as possible there is a part of the dividend left over, this number is called the **Remainder.**

Note. The remainder will of course be less than the divisor.

42. Division Table. Although the Multiplication Table is sufficient for the operation of division, the Division Table is given for those who may prefer to use it.

DIVISION TABLE.

1 is contained in	2 is contained in	3 is contained in	4 is contained in	5 is contained in	6 is contained in
1 .. 1	2 .. 1	3 .. 1	4 .. 1	5 .. 1	6 .. 1
2 .. 2	4 .. 2	6 .. 2	8 .. 2	10 .. 2	12 .. 2
3 .. 3	6 .. 3	9 .. 3	12 .. 3	15 .. 3	18 .. 3
4 .. 4	8 .. 4	12 .. 4	16 .. 4	20 .. 4	24 .. 4
5 .. 5	10 .. 5	15 .. 5	20 .. 5	25 .. 5	30 .. 5
6 .. 6	12 .. 6	18 .. 6	24 .. 6	30 .. 6	36 .. 6
7 .. 7	14 .. 7	21 .. 7	28 .. 7	35 .. 7	42 .. 7
8 .. 8	16 .. 8	24 .. 8	32 .. 8	40 .. 8	48 .. 8
9 .. 9	18 .. 9	27 .. 9	36 .. 9	45 .. 9	54 .. 9
10 .. 10	20 .. 10	30 .. 10	40 .. 10	50 .. 10	60 .. 10
11 .. 11	22 .. 11	33 .. 11	44 .. 11	55 .. 11	66 .. 11
12 .. 12	24 .. 12	36 .. 12	48 .. 12	60 .. 12	72 .. 12

7 is contained in	8 is contained in	9 is contained in	10 is contained in	11 is contained in	12 is contained in
7 .. 1	8 .. 1	9 .. 1	10 .. 1	11 .. 1	12 .. 1
14 .. 2	16 . 2	18 .. 2	20 .. 2	22 .. 2	24 .. 2
21 .. 3	24 .. 3	27 .. 3	30 .. 3	33 .. 3	36 .. 3
28 .. 4	32 .. 4	36 .. 4	40 .. 4	44 .. 4	48 .. 4
35 .. 5	40 .. 5	45 .. 5	50 .. 5	55 .. 5	60 .. 5
42 .. 6	48 .. 6	54 .. 6	60 .. 6	66 .. 6	72 .. 6
49 .. 7	56 .. 7	63 .. 7	70 .. 7	77 .. 7	84 .. 7
56 .. 8	64 .. 8	72 .. 8	80 .. 8	88 .. 8	96 .. 8
63 . 9	72 .. 9	81 .. 9	90 .. 9	99 .. 9	108 .. 9
70 .. 10	80 .. 10	90 .. 10	100 .. 10	110 .. 10	120 .. 10
77 .. 11	88 .. 11	99 .. 11	110 .. 11	121 .. 11	132 .. 11
84 .. 12	96 .. 12	108 .. 12	120 .. 12	132 .. 12	144 .. 12

43. Sign of Division. The sign ÷ (by, or divided by,) means that the number which precedes the sign is to be divided by that which follows it.

Thus $18 \div 9 = 2$ means that 18 divided by 9 gives quotient 2.

Again, $(7 + 5 + 6) \div 3 = 6$ means that when the number, which is the result of $7 + 5 + 6$, viz., 18, is divided by 3, the quotient is 6.

So $24 \div (8 - 2) = 4$ means that 24 divided by the result of $8 - 2$, viz., 6, gives quotient 4.

---

44. There are two different kinds of questions in Division.

EXAMPLE 1. How many times is 7 contained in 56?

By the Multiplication Table we know that 8 times 7 make 56.

Or, by the Division Table, we know that 7 is contained in 56 8 times.

∴ the result is 8 times.

EXAMPLE 2. If 54 be divided into 6 equal parts, how many units will there be in each part?

To put 1 in each of the 6 parts 6 must be taken from the 54. Hence the question becomes, "How many times is 6 contained in 54?" From either table we know that the number of times is 9; ∴ there will be nine units in each part.

Note In all questions like Example 1, the divisor being a part of the dividend, will have the same unit, and the unit of the quotient is abstract.

In all questions like Example 2, the divisor is abstract or a mere number, and the unit of the quotient, which is a part of the dividend, is the same as that of the dividend.

The operation deals with numbers only, and the unit of the result is known from the nature of the question.

Example 3. How many pencils costing 5 cents each can be bought for 60 cents?

Since 5 is contained 12 times in 60, the result is 12 pencils.

Example 4. If 72 nuts be divided equally among 8 boys, how many will each get?

To give 1 nut to each boy 8 nuts will be required; 8 is contained 9 times in 72; ∴ the result is 9 nuts.

EXERCISE XVIII.

1. How many boys can receive 3 apples each out of 24 apples?

2. If 36 boys walk four abreast, how many ranks will there be?

3. How many pounds of sugar costing 12 cents a pound can be bought for 100 cents; and how many cents will there be left over?

4. A week is 7 days. How many weeks in 42 days?

5. A basket contains 72 eggs. How many times can 6 eggs be taken out of it?

6. If a boy walk 5 miles a day, in how many days will he walk 40 miles?

7. How many pages a day must a person read to finish 132 pages in 12 days?

8. Divide 63 cents by 9, that is, divide 63 cents into 9 equal parts, and find how many there are in each part?

9. What is the eighth part of 48 cents?

10. Divide 84 cents by 12 cents, that is, find how many times 12 cents are contained in 84 cents.

45. To divide by a number not greater than 12.

Example 1. Divide 876 by 3. In other words, find how many times 3 can be subtracted from 876.

3)876	200
600	
—	
276	90
270	
—	
6	2
6	—
	292

Explanation. In 800, 3 is contained 200 times: subtract 200 times 3 or 600, and the remainder is 276. Then in 276, 3 is contained 90 times: subtract 90 times 3, or 270. Then in the 6 remaining, 3 is contained 2 times: subtract 6, and there is no remainder. Altogether, 2 has been subtracted 292 times.

SHORT FORM.

3)876
———
292

Explanation. In 8 (800) 3 is contained 2 (200) times, with remainder 2 (200): set down 2 under 8 and carry 2 hundreds, or 20 tens. In 27 (tens) 3 is contained 9 (90) times: set down 9 under the 7. In 6, 3 is contained 2 times: set down 2 under the 6.

Example 2. Divide 4176 by 5.

5)4176
———
835—1

Explanation. In 41, 5 goes 8 times with 1 over. In 17, 5 goes 3 times with remainder 2. In 26, 5 goes 5 times with remainder 1.

EXERCISE XIX.

1. 2)6842
2. 2)7654
3. 2)4798
4. 3)3696
5. 3)4569
6 3)7518
7. 4)8408
8. 4)9216
9. 4)5768

10. 5)5100
11. 5)7530
12. 5)8645
13. 6)6924
14. 6)8106
15. 7)7154
16. 7)8358
17. 8)1032
18. 8)1168
19. 9)9180
20. 9)7398
21. 10)8670
22. 10)3768
23. 11)8162
24. 11)5876
25. 12)1584
26. 12)10452
27. 12)3789

28. How many 5 cent pieces will make up 375 cents?

29. How many steps, 2 feet long, will a boy take in walking a mile (5280 feet)?

30. If a boy read 7 pages a day, how long will it take him to read a book containing 448 pages?

31. If a man earns 4 dollars in 3 days, how many dollars will he earn in 312 days?

32. Divide sixty thousand four hundred and one by eleven.

33. How many times can 12 be subtracted from fifteen thousand and twenty-four?

34. If 5 apples are sold for a cent, how much will it cost to give 3 apples each to 175 children?

35. How many pounds of sugar, worth 9 cents a pound, should be given for 6 bushels of barley, worth 78 cents a bushel?

36. If A can walk 9 feet while B walks 8 feet, and they set out together to walk in the same direction, how far ahead will A be when B has walked a quarter of a mile (1320 feet)?

37. A and B are a mile (5280 feet) apart. If they set out at the same time and A walk 6 feet a second and B 5 feet, how long will it take them to meet?

46. Rule for Simple Division, when the Divisor is a number not larger than 12.

Place the divisor and dividend thus :

$$\text{divisor}) \underline{\text{dividend.}}$$

Take off from the left hand of the dividend the least number of figures which will make a number not less than the divisor.

Find by the Multiplication Table how often the divisor is contained in this number ; write the quotient under the units' figure of this number, and take notice of the remainder, whether it be any number or 0.

With the remainder (whether it be any number or 0), take the least number of the figures next following in the dividend which will, affixed to the remainder, make a number not less than the divisor. Proceed, as above, with this new dividend to find the next figure of the quotient ; taking care to place after the first figure in the quotient a cipher for every figure just brought down from the dividend except the last.

Continue this process till all the figures in the dividend have thus been brought down.

Note. If there is a remainder left at the end of the operation, set it apart to the right of the quotient.

EXAMPLE 1. Divide 87162 by 3.

$$\begin{array}{r} 3)87162 \\ \hline 29054 \end{array}$$

Explanation. In 8 (80000) 3 is contained 2 (20000 times), with 2 (20000) over. In 27 (27000) 3 is contained 9 (9000 times). In 1 (100) 3 is contained 0 (no hundreds), with 1 (100) over. In 16 (160) 3 is contained 5 (50 times), with 1 (10) over. In 12, 3 is contained 4 times.

EXAMPLE 2. Divide 10680158 by 12.

$$\begin{array}{r} 12\overline{)10680158} \\ \hline 890013 \ldots 2 \end{array}$$

Explanation. 12 into 106 goes 8 times and 10 over. 12 into 108 goes 9 times. 12 into 0 goes 0. 12 into 1 goes 0 and 1 over. 12 into 15 goes 1 time and 3 over. 12 into 38 goes 3 times and 2 over.

47. TEST OF CORRECTNESS. The quotient multiplied by the divisor, the remainder, if any, being added, should produce the dividend.

EXERCISE XX.

Divide

1. 316 by 2
2. 528 by 3
3. 1352 by 4
4. 6785 by 5
5. 3750 by 6
6. 5383 by 7
7. 15376 by 8
8. 21654 by 9
9. 36870 by 10
10. 41932 by 11
11. 91476 by 12
12. 49006 by 8
13. 9350 by 12
14. 17069 by 7
15. 25000 by 9
16. 31758 by 6
17. 11033 by 3
18. 4968 by 10
19. 999999 by 11
20. 123456 by 5
21. 800000 by 9
22. 700000 by 9

23. Divide forty thousand and twelve by 7.

24. Divide the sum of 28 and 36 by their difference.

25. How many times is twelve contained in the sum of the first 6 odd numbers ?

26. How many times can 11 be subtracted from 507099 ?

27. What number must be subtracted from 10000 so that the remainder may be exactly divisible by 12 ?

28. What is the least number which must be added to 73068 to make the sum exactly divisible by 8 ?

29. A farmer sells 6 dozen eggs at 24 cents a dozen and buys sugar at 9 cents a pound. How many pounds can he buy?

30. A dairyman exchanges with a shop-keeper 17 pounds of butter at 25 cents a pound for calico at 11 cents a yard. How many yards will he get, and how many cents over?

---

48. Division by Factors.

To divide by a number which can be broken up into factors not greater than 12.

Example 1. Divide 690 by 15.

5)690
3)138
46

Explanation. Since 15 = 3 × 5, we divide 690 by 5 and find that 690 contains 138 fives. Then dividing 138 by 3, we find that 3 fives are contained in 138 fives 46 times.

Example 2. Divide 3176 by 42.

	Rem.
7)3176	
6)453...5	5
75...3	21
	26

Explanation. 42 = 6 × 7. Dividing by 7, we find 453 sevens and 5 units over. Then dividing 453 sevens by 6 sevens we get quotient 75 and 3 sevens, or 21, over. The whole remainder is therefore the sum of 21 and 5, that is, 26.

Example 3. Divide 37508 by 135.

5)37508	
9)7501...3	3
3)833...4 × 5	20
277...2 × 9 × 5	90
	113

Explanation. 135 = 3 × 9 × 5; ∴ we divide by 5, 9 and 3. The first remainder is 3 units; the second remainder is 4 fives, or 20; and third remainder is 2 × 45, or 90; ∴ the whole remainder is the sum of 3, 20 and 90, which is 113.

Note. The remainder left by any divisor must be multiplied by all the preceding divisors.

49. To divide by 10, 100, 1000 and so on.

Example 1. Divide 635 by 10.

10)635
63...5

Explanation. We see that this might be done by simply cutting off the last figure for the remainder, and taking the other figures as the quotient.

Example 2. Divide 3147 by 100.

10)3147
10)314...7 7
31...4 × 10 40
47

Or more briefly,

100)31,47
31...47

Explanation. The operation is performed briefly by cutting off the last two figures for the remainder, and taking the rest for the quotient.

Example 3. Divide 17854 by 1000.

10)17854
10)1785...4 4
10)178...5 × 10 50
17...8 × 100 800
854

Or more briefly,

1000)17,854
17...854

Explanation. We cut off the last three figures for the remainder, and the rest will give the quotient.

Note. The same method may be used for any number of tens as factors.

Example 4. Divide 76135 by 2400.

100)761,35
6)761...35 — 35
4)126... 5 × 100 — 500
31... 2 × 600 — 1200
1735

Or more briefly,

600)761,35
4)126 ... 535
31...1200
1735

Example 5. Resolve 360 into its prime factors.

2)360
2)180
2)90
3)45
3)15
5

Explanation. We divide by 2 as many times as we can without remainder, then by 3 in the same way.

Hence 360 = 2 × 2 × 2 × 3 × 3 × 5.

Note 1. We must try the prime numbers 2, 3, 5, 7, etc., dividing as often as possible by each. The operation will terminate as soon as the quotient is a prime number.

Note 2. Any even number is divisible by two, and any number ending in 5 is divisible by 5. Any number ending in 0 is divisible by 2 and by 5.

EXERCISE XXI.

Divide

1. 5935 by 15
2. 17[illegible]98 by 18
3. 73416 by 24
11. 713564 by 63
12. 85946 by 48
13. 470038 by 135

4. 60032 by 28
5. 91800 by 72
6. 100044 by 84
7. 813708 by 108
8. 287617 by 121
9. 350640 by 144
10. 173052 by 198
14. 95476 by 96
15. 580475 by 147
16. 613948 by 216
17. 39654 by 600
18. 843717 by 800
19. 150493 by 420
20. 931875 by 56000

21. Break up 225 into its prime factors.

22. Resolve 625 into its prime factors.

23. Find what prime numbers multiplied together will make 105.

24. Express 715 as the continued product of its prime factors.

---

50. To divide by any number greater than 12.

Example 1. Divide 41936 by 23.

$$\begin{array}{r|r} 23)41936 & 1000 \\ 23000 & \\ \cline{1-1} 18936 & 800 \\ 18400 & \\ \cline{1-1} 536 & 20 \\ 460 & \\ \cline{1-1} 76 & 3 \\ 69 & \\ \hline 7 & 1823 \end{array}$$

Explanation. In 41 thousand 23 is contained more than 1 thousand times, but less than 2 thousand times. Subtract 1000 times 23, or 23000 from 41936 and the remainder is 18936. In 189 hundreds 23 is contained more than 8 hundred times, but less than 9 hundred times. Subtract 800 times 23, or 18400, from 18936 and the remainder is 536. In 53 tens 23 is contained more than 2 times ten, but less than 3 times ten. Subtract 20 times 23, or 460, from 536 and the remainder is 76. In 76, 23 is contained more than 3 times, but less than 4 times. Subtract 3 times 23, or 69, and only 7 remains. Altogether, 23 is contained 1823 times, with 7 over.

The above operation is usually shortened by omitting ciphers and by bringing down only that part of the dividend which is required at each step. Thus :

```
23)41936(1823
   23
   ——
   189
   184
   ———
    53
    46
    ——
     76
     69
     —
      7
```

Explanation. We say, in 41, 23 is contained 1. Subtract 23 from 41 and the remainder is 18. Bring down 9. In 189, 23 is contained 8 times. Subtract 8 times 23, or 184, from 189, and the remainder is 5. Bring down 3. Then in 53, 23 is contained 2 times. Subtract twice 23, or 46, from 53 and the remainder is 7. Bring down 6. Then in 76, 23 is contained 3 times. Subtract 3 tlmes 23, or 69, and the remainder is 7. Thus, altogether, the quotient is 1823, and the remainder 7.

EXAMPLE 1. Divide 1144704 by 176.

```
176)1144704(6504
    1056
    ————
     887
     880
     ———
      704
      704
      ———
```

Explanation. We must take 1144 to get a number not less than 176. In 1144 we find by trial that 176 goes 6 times. Subtract 6 times 176, or 1056, from 1144 and the remainder is 88. Bring down 7. Then in 887 we find by trial that 176 goes 5 times. Subtract 5 times 176, or 880 from 887. To the remainder 7, affix 0. Then in 70 176 goes 0 times. Set 0 in the quotient and bring down 4. Then we find by trial that 176 goes into 704, 4 times. Subtract 4 times 176, or 704, and 0 remains. Altogether, the quotient is 6504.

EXERCISE XXII.

Divide

1. 1534 by 13
2. 2091 by 17

11. 27540 by 85
12. 10682 by 98

3. 2356 by 19
4. 5129 by 23
5. 4092 by 31
6. 48359 by 37
7. 9798 by 46
8. 13572 by 58
9. 20020 by 65
10. 31844 by 76
13. 36855 by 105
14. 46980 by 116
15. 78936 by 138
16. 35875 by 175
17. 9612 by 267
18. 19345 by 365
19. 100744 by 508
20. 500042 by 761

21. How many times can 29 be subtracted from 3016?

22. Divide ten thousand and thirty-two by thirty-eight?

23. What number multiplied by 57 will produce 28,956?

24. Find the value of 98072 ÷ 13.

25. Divide the sum of 4386 and 4352 by their difference.

26. Find the value of (12586 × 6807) ÷ (319 − 276).

27. The dividend is 65124 and the divisor is 67. What is the quotient?

28. How many bushels of wheat at 95 cents a bushel are worth 665 bushels of barley at 65 cents a bushel?

29. If a train makes 45 miles an hour, how long will it take to go 855 miles?

30. If a farmer grows 1036 bushels of wheat on 37 acres, how many bushels is that per acre?

31. A driver sold 376 sheep for 1950 dollars, thus gaining 446 dollars. How much a head did he pay for them ?

51. Rule for Division, when the Divisor is a number greater than 12.

RULE. Place the divisor and dividend thus :

divisor)dividend(

leaving a space for the quotient on the right of the dividend.

Take off from the left hand of the dividend the least number of figures which make a number not less than the divisor.

Find how many times the divisor is contained in this number ; write the quotient as the left-hand figure of the whole quotient ; multiply the divisor by this figure, and place the product under the number taken off from the left of the dividend, and subtract.

On the right of the remainder (whether it be any number or 0) place the least number of figures next following in the dividend which will, affixed to the remainder, make a number not less than the divisor. Proceed as above with this new dividend to find the next figure of the quotient ; taking care to place after the first figure in the quotient a cipher for every figure just brought down from the dividend except the last.

Continue this process till all the figures in the dividend have thus been brought down.

If there be a number left at the end of the operation, it is the remainder.

Note. If any remainder be equal to or greater than the divisor, the last figure of the quotient must be changed for one greater.

Example 1. Divide 244265850 by 813.

```
813)244265850(300450
    2439
    ————
       3658
       3252
       ————
        4065
        4065
        ————
```

Explanation. The least part of the dividend not less than 813 is 2442. In this number 813 is contained 3 times. Set 3 in the quotient and subtract 3 times 813, or 2439, from 2442. To the remainder 3 affix 6, and as 36 does not contain 813, set 0 in the quotient and affix 5 to 36. Since 365 does not contain 813, set another 0 in the quotient and affix 8 to 365. In 3658, 813 is contained 4 times. Set 4 in the quotient and subtract 4 times 813, or 3252, from 3658. To the remainder 406 affix 5. In 4065, 813 is contained 5 times. Set 5 in the quotient and subtract 5 times 813, or 4065, from 4065. As there is another figure to bring down, which does not contain 813, set 0 in the quotient.

Example 2. Divide 718689 by 6800.

```
68)7136,89(104
   68
   ———
    336
    272
    ———
     6489
```

Explanation. Since 6800 = 100 times 68, we first divide by 100; which is done by cutting off two figures, giving quotient 7136 and remainder 89. Dividing 7136 by 68, we get quotient 104 with remainder 64 hundreds. Adding the former remainder, 89, the remainder is 6489.

We might have divided by 6800 in the usual way, but the above method is shorter.

EXERCISE XXIII.

1. 69480 by 18
2. 12376 by 26
3. 40749 by 47
4. 616850 by 730

11. 10368280 by 785
12. 9051874 by 9870
13. 6389075 by 1658
14. 1358794 by 6078

5. 400000 by 950
6. 1054053 by 117
7. 1368620 by 146
8. 5164236 by 378
9. 2634632 by 517
10. 2317564 by 804
15. 3185647 by 9367
16. 4250000 by 6060
17. 5146387 by 8976
18. 1157854 by 1758
19. 68761000 by 9400
20. 9250867 by 8700

21. Divide sixty thousand five hundred and eighty by nine hundred and six.

22. Divide the product of 85 and 132 by 187.

23. How many times does 6351 contain 73 ?

24. By what number must 65 be multiplied to produce 9490 ?

25. Find the value of $(41610 + 41534) \div (41610 - 41534)$.

26. A bushel of barley weighs 48 pounds. How many bushels are there in a car-load weighing 25,536 pounds ?

27. What other number besides 37 will divide 6401 without remainder ?

28. If a clerk can count 135 notes in a minute, in what time can he count 81,000 dollars in five dollar notes ?

29. What number, less than 29, must be subtracted from 5378 to obtain a number exactly divisible by 29 ?

30. A bag of nuts supplied 20 nuts each to 49 boys and 6 nuts over. How many boys could have each received 17 nuts ?

31. A bushel of oats weighs 34 pounds, a bushel of wheat 60 pounds. How many bushels of oats will weigh as much as 119 bushels of wheat ?

32. If a steamship goes 17 miles an hour, how many days of 24 hours will she take to sail 2,856 miles ?

33. A mile is 5280 feet. How many rails 19 feet long will be required to lay a single track from Toronto to Collingwood, 95 miles?

34. Find the number nearest to 10,000 which is exactly divisible by 67; also that which is divisible by 670.

35. A man's income being 882 dollars a year, how much can he spend each week so as to save 50 dollars yearly?

36. When 17 is added to a certain number, 71 is contained in the sum 83 times. Find the number.

37. Find the least number which must be added to 3000 to make a number exactly divisible by 67. Also find the least number which must be added to 3000 to form a number divisible by 46.

38. Taking the distance between Toronto and Montreal as 336 miles, where will an express train from Montreal, going 40 miles an hour, meet a freight train which started at the same time from Toronto and runs 16 miles an hour?

93. There are 3564 papers to be read by three examiners. The first reads 73 papers a day, and the second 67; how many must the third read daily that the work may be finished in 18 days?

40. *A* sets out two days before *B* and walks 24 miles a day. How fast must *B* travel to overtake *A* in four days?

41. Taking the distance between Montreal and Toronto as 336 miles, if a freight train from Toronto going 18 miles an hour meets the express from Montreal 8 hours after starting, when will the express reach Toronto?

42. A horse and carriage cost 528 dollars, the horse with the harness cost 366 dollars, and the carriage and harness together cost 482 dollars; what did each cost?

42. How long will a train 330 feet long, which goes 55 feet a second, be heard crossing a bridge whose length is 715 feet?

52. The divisor, dividend, quotient and remainder are connected in the following ways:

(1) Divisor = (dividend − remainder) ÷ quotient.

(2) Dividend = divisor × quotient + remainder.

(3) Quotient = (dividend — remainder) ÷ divisor.

(4) Remainder = dividend — divisor × quotient.

Hence, if any three of the above numbers be given we can find the fourth.

Example 1. What number must be subtracted from 135 to get a number in which 12 shall be contained 11 times?

$$\begin{aligned}\text{Remainder} &= 135 - 12 \times 11\\ &= 135 - 132 = 3.\end{aligned}$$

Example 2. What number contains 8 seven times with remainder 5?

$$\begin{aligned}\text{Dividend} &= 8 \times 7 + 5\\ &= 56 + 5 = 61.\end{aligned}$$

EXERCISE XXIV.

1. The dividend is 378, the divisor 25, and the quotient 15; find the remainder without dividing.

2. What number divided by 67 will give quotient 34 and remainder 26?

3. If 4173 be divided by a certain number, the quotient is 112, and the remainder 29. Find the number.

4. How many times must 12 be taken to make up 5184?

5. If the product of 83 and 61 be divided by 83, what will be the quotient? What if it be divided by 61?

6. A number of nuts being divided among 13 boys, each got 17 nuts and there were 8 over. If there had been 17 boys how many would each have got?

7. How much must be subtracted from 3579 that the remainder may contain 123 exactly 112 times?

8. What number must be added to 5005 to make 47 times 107?

Miscellaneous Examples.

PAPER I.

1. What is an addend? What is a sum? How do you write the addends to get their sum?

2. Find the sum of seven hundred and nineteen thousand and two, five million thirty-one thousand and ninety, four hundred and nine million thirty-one thousand one hundred.

3. Multiply 59730215 by 7016, and divide 732540408423 by 273.

4. Add together ten thousand and ninety, one million one thousand and one, nine hundred thousand one hundred; and subtract twenty thousand two hundred and ten from the sum. Express the answer in words.

5. Multiply 98500 by 9070; and divide 62501 by 250.

6. Supposing a labourer to receive $12 per week, find what his wages will amount to for a year.

PAPER II.

1. Give the meaning of the following words: Multiplicand, Multiplier, Product. What are the multiplier and multiplicand with respect to the product?

2. A person rides a distance of 78 miles in 13 hours. How long will it take him, travelling at the same rate, to finish a journey of 60 miles?

3. Add together fourteen million eight thousand and fifty, one hundred and three thousand and nine; from the sum subtract two million twenty thousand and eighty-seven, and write down the result in words.

4. Divide 275487 by 736, and prove the correctness of the result.

5. Express the date of the present year of our Lord (1882) in words, in the usual notation, and in Roman numerals.

6. The population of Canada in 1871 was 3680596, and 4352080 in 1881. Find (1) the total increase, and (2) the average annual increase in the population for the time between these two dates.

PAPER III.

1. What is the meaning of each of the words: Dividend, Divisor, Quotient? How can you find the dividend when there is a remainder?

2. The number of votes polled by two candidates at an election was 2163, and the majority of the successful candidate was 195. What number of votes did each candidate receive?

3. Express trains run 42 miles an hour. Find the time required for a body travelling with the speed of such a train to reach the sun, the distance of the sun from the earth being 91430000 miles.

4. The product of two numbers is 15580656, and one of them is 6552. Find the other.

5. Express in words and in figures how much greater the value of one 5 is than the other in the number 658757.

6. Multiply 2468 by 3057; and divide the product by 3702.

PAPER IV.

1. Explain how is it that all questions in Multiplication can be done by Addition, but all questions in Addition cannot be done by Multiplication?

2. Find the sum, difference, product and quotient of 537152 and 763.

3. Find the number which subtracted from 80000 leaves 57735; and divide it by 365.

4. Twenty-five years ago a man was four times as old as his son, whose present age is 33. What is the present age of the father?

5. Add together the following numbers:—Three hundred and forty thousand and fifty, five million nine hundred and twenty-two thousand and nine, seven hundred and four thousand three hundred and four, twenty thousand and five, sixty-five thousand, and six hundred. Subtract from the sum three hundred and ninety-seven thousand eight hundred and ninety-seven.

6. In building a viaduct of one hundred and one arches, ninety thousand and nine bricks are used for each arch. Find the number of bricks used.

PAPER V.

1. If 21 sovereigns are equal in value to 20 guineas, how many sovereigns are equivalent to 8340 guineas?

2. A man counted on a hundred acres 476 oak trees, 1968 maple, 4072 beech, 902 hemlock, 127 elm, and 97 ash. How many trees altogether?

3. Add 6543, 807, 54091, 9999, and 38.

4. How many strokes does a clock telling the hours strike in twelve hours?

5. A train consists of seven carriages, in which are the following numbers of persons : 28, 31, 35, 11, 22, 18, 21. If 21 persons now get into each of the carriages, how many passengers will there be altogether?

6. The number of passengers carried in the year ending June, 1880, by the Grand Trunk was 1995526; by the Great Western, 1241111 ; by the Northern, 386169 ; by the Intercolonial, 581483 ; by the Canada Southern, 278818 ; by the Midland, 116857 ; by the Toronto, Grey and Bruce, 121685 ; and by the Toronto and Nipissing, 89933. What was the total number of passengers?

PAPER VI.

1. Obtain the sum of forty-six times seven thousand and twenty, seventeen times one million and one, and thirty-three times thirty-three.

2. Subtract two thousand and fifty from forty-one thousand and thirty-three; then to the remainder add sixteen thousand five hundred and seventy-two.

3. Show that the sum of 81459 and 54306 is equal to five times their difference.

4. In 1871 the population of Prince Edward Island was 94021 ; of Nova Scotia, 387800 ; of New Brunswick, 285594 ; of Quebec, 1191516 ; of Ontario, 1620851 ; of Manitoba, 12728 ; of British Columbia, 33586 ; of North-West Territories, 60500. What was the total population?

5. In 1881 the population of Prince Edward Island was 108928 ; of Nova Scotia, 440585 ; of New Brunswick, 321129 ; of Quebec, 1358469 ; of Ontario, 1913460 ; of Manitoba, 49509 ; of British Columbia, 60000 ; of North-West Territories, 100000. Find whole population of Canada in 1881.

6. From the preceding two questions, find the increase in the population of Canada from 1871 to 1881.

SECTION II.

DECIMAL MONEY.

53. Decimal Money is money in which the unit of value is subdivided in a tenfold manner. Thus, in Canada and in the United States, $1 = 10 ten-cent pieces (dimes) = 100 cents. In France 1 franc = 10 dimes = 100 cents.

The advantage of this system is that numbers denoting money can be written like simple numbers, and the rules of Addition, Subtraction, Multiplication and Division, already explained, will apply to them.

Thus 4 dollars, 6 ten-cent pieces and 7 cents = 400 cents + 60 cents + 70 cents = 467 cents.

So 785 cents = 7 dollars + 8 ten-cent pieces + 5 cents, or = 7 dollaɹs and 85 cents.

The symbol for dollar is $, and that for cent is c.

A full stop, called the decimal point, is used to separate the dollars from the cents.

Thus 656 cents is written $6.56, and 500 cents is written $5.00.

Hence, to reduce dollars and cents to a simple number of cents, we have merely to remove the decimal point two places to the right.

Thus, $12.48 = 1248 c., and $15, or $15.00, = 1500 c.

Again, to reduce cents to dollars and cents, we have only to cut off the last two figures by the decimal point.

Thus, 185 c. = $1.85, and 200 c. = $2.00.

54. The way to apply the four simple rules to numbers denoting money will be seen from the following examples :

EXAMPLE 1. Find the sum of 47c., 86c., 35c. and 76c.

$$\begin{array}{r} 47 \\ 86 \\ 35 \\ 76 \\ \hline 244\text{c.} = \$2.44. \end{array}$$

EXAMPLE 2. Add together \$35, \$47, \$9 and \$367 :

$$\begin{array}{r} \$\ 35 \\ 47 \\ 9 \\ 367 \\ \hline \$458 \end{array}$$

EXAMPLE 3. A person spent \$4.65 on groceries, \$3.87 on meat, \$1.36 on vegetables, and 84c. on bread. How much did he spend altogether ?

$$\begin{array}{r} 465 \\ 387 \\ 136 \\ 84 \\ \hline 1072\text{c.} = \$10.72. \end{array} \qquad \text{or,} \qquad \begin{array}{r} \$\ 4.65 \\ 3.87 \\ 1.36 \\ .84 \\ \hline \$10.72 \end{array}$$

EXAMPLE 4. A storekeeper took in during the day \$137.28, and paid out \$52.75 ; by how much was his money increased ?

$$\begin{array}{r} 13728 \\ 5275 \\ \hline 8453\text{c.} = \$84.53 \end{array} \qquad \text{or,} \qquad \begin{array}{r} \$137.28 \\ 52.75 \\ \hline \$84.53 \end{array}$$

Example 5. A person bought a pound of tea for 85c , a pound of coffee for 45c, a bag of flour for $3.28, and a box of starch for 90c., and gave a $10 note in payment; how much did he receive in change?

$$\begin{array}{r} 85 \\ 45 \\ 328 \\ 90 \\ \hline 548 \end{array} \qquad\qquad \begin{array}{r} 1000 \\ 548 \\ \hline 452 \end{array} = \$4.52.$$

Example 6. A butcher bought 8 sheep at $6.75 apiece, how much did he pay for the lot?

$$\begin{array}{r} 675 \\ 8 \\ \hline 5400\text{c.} \end{array} = \$54.00 \qquad\qquad \text{or,}\quad \begin{array}{r} 6.75 \\ 8 \\ \hline \$54.00 \end{array}$$

Example 7. Find the cost of 46 barrels of flour at $6.38 a barrel.

$$\begin{array}{r} 638 \\ 46 \\ \hline 3828 \\ 2552 \\ \hline 29348\text{c.} \end{array} = \$293.48 \qquad\qquad \text{or,}\quad \begin{array}{r} 6.38 \\ 46 \\ \hline 38.28 \\ 255.2 \\ \hline \$293.48 \end{array}$$

Example 8. $257.25 was equally divided between 7 men, how much did each get? See 44, p. 41.

$$\begin{array}{r} 7)25725 \\ \hline 3675\text{c.} \end{array} = \$36.75 \qquad\qquad \text{or,}\quad \begin{array}{r} 7)257.25 \\ \hline \$36.75 \end{array}$$

Example 9. A man earned $1445.60 in a year (52 weeks), how much was that for each week?

```
52)144560(2780c. = $27.80     (44)
   104
   ---
    405
    364
    ---
     416
     416
     ---
```

Example 10. How many bushels of wheat at $1.36 a bushel can be bought for $1000?

```
136)100000(735 bushels     (44)
    952
    ---
     480
     408
     ---
      720
      680
      ---
       40c. over
```

Example 11. A farmer exchanged 36 bushels of barley at 98c. a bushel for oats at 42c. a bushel, how many bushels did he get?

```
   98
   36
  ---
  588
 294
 ----
42)3528(84 bushels.
   336
   ---
    168
    168
```

Or more briefly,

$$\frac{\overset{14}{\cancel{98}} \times \overset{6}{\cancel{36}}}{\underset{7}{\cancel{42}}} = 84 \text{ bushels.}$$

55. In the shorter method the multiplication is indicated by the sign × and the division by writing the divisor under the dividend, with a line between. Then any factor common to the dividend and divisor is struck out (cancelled). Thus 36 and 42 are both divided by the common factor 6, and 98 and 7 by 7, the 1 under 7 being omitted ; the result is 14 × 6 or 84.

EXERCISE XXV.

1. Find the sum of $7.65, $19.84, $36.58, $42, $1.75 and $8.

2. Find the sum of $3.42, $75, 85 cents, $13.50, $9.76, $0.58 and $0.09.

3. How much does $13.27 exceed $9.52 ?

4. If a person having $10 spend $1.55, $1.68, $2.36 and 75 cents, how much will he have left ?

5. Find the cost of 9 chairs at $1.75 each.

6. Find the value of 47 bushels of wheat at $1.38 a bushel ?

7. If a man who earns $15 a week gives 25 cents at church on Sunday and spends $1.75 on each of the other 6 days, how much will he have left ?

8. A person bought a box of eggs, 36 dozen, at 18 cents a dozen, and paid, besides, 24 cents for the box and 30 cents for freight. Find how much he paid altogether, and how much a dozen the eggs cost him ?

9. A farmer sells a hog to a grocer for $6, and buys of him a pound of tea at 65 cents, 2 pounds of coffee at 36 cents a pound, and 12 pounds of sugar for a dollar ; how much money will he receive ?

10. A distiller sells 3 barrels of whiskey, each containing 60 gallons, at $1.05 a gallon; how many bushels of

barley, at 75 cents a bushel, can he buy with the money received?

11. A lady buys 3 yards of cloth at 88 cents a yard, 17 yards of calico at 12 cents a yard, and 4 pairs of socks at 45 cents a pair. How much change will she receive out of a $10 note?

12. A clerk paid $3.75 a week for his board, and his other expenses for the year were $100. How much could he save out of his salary of $400 a year.

13. A man earned $10 a week and put by during the year $200.20. How much a week did he spend?

14. A farmer exchanged 54 bushels of oats at 52 cents a bushel for wheat at $1.17 a bushel. How many bushels of wheat did he get?

15. A gentleman at his death left $65,625; one-third to go to his widow, and the rest to be equally divided among his 3 sons and 2 daughters. How much did each child receive?

16. A person paid for a tub of butter $9.12, which sum included 35 cents for freight and 60 cents for the tub. The butter cost originally 19 cents a pound. How many pounds were there?

17. If a sovereign be worth $4.87, how nearly can $1000 be paid in sovereigns?

18. If a workman make each working day two trips on the street railway, how much will he save in a year (52 weeks) by buying 25 tickets for a dollar instead of paying 5 cents each trip?

19. *A* has only 25 cent pieces and *B* only 20 cent pieces, how can *A* pay *B* $3.20 for a hat?

SECTION III.

THE UNITARY METHOD.

56. The Unitary Method of solving problems is so named because the first step of the solution is to find the value, or result, of the unit.

Example 1. If 3 pounds of sugar cost 24 cents, what will be the value of 5 pounds?

The value of 3 pounds = 24 cents,
∴ " " 1 pound = 24 ÷ 3 or, 8 cents.
∴ " " 8 pounds = 8 × 5, or 40 cents.

Example 2. If a man earn $2 in 6 hours, in what time will he earn $20?

$2 is earned in 6 hours,
∴ $1 " 6 ÷ 2, or 3 hours,
∴ $20 " 3 × 20, or 60 hours.

Example 3. If the wages of 6 men for a week be $45, what will be the wages of 20 men for the same time?

The wages of 6 men = $45 See 55, p. 66.
∴ The " of 1 man = $\$\frac{45}{6}$
∴ The " of 20 men = $\$\frac{\overset{15}{\cancel{45}} \times \overset{10}{\cancel{20}}}{\underset{\cancel{3}}{\cancel{6}}} = \$150.$

Note 1. In the above examples the value, or result, of a unit has been found by division, but there is another class of questions in which the result of a unit is found by multiplication.

Example 4. If 15 men can build a house in 4 weeks, how many men will be required to build it in 3 weeks?

To build it in 4 weeks requires 15 men.
∴ To " " 1 week " 15 × 4 or 60 men.
∴ To " " 3 weeks " 60 ÷ 3 or 20 men.

Example 5. If a steamship, which goes 15 miles an hour, takes 8 days to cross the Atlantic Ocean, what speed must a vessel have to cross in 5 days?

To cross in 8 days requires 15 miles an hour.
∴ To " in 1 day " 15 × 8 " "
3
∴ To " in 5 days " $\frac{\cancel{15} \times 8}{\cancel{5}}$ or 24 miles.

Note 2. The pupil must be careful to place the number which is of the same kind as the required number to the right hand, and to consider whether the result of the unit is to be found by multiplication or by division.

EXERCISE XXVI.

1. If 12 oranges cost 24 cents, how many will 10 cents buy?

2. If 12 oranges cost 24 cents, how much must be paid for 8 oranges?

3. If 2 men can saw a cord of wood in 6 hours, how long will it take 3 men to saw a cord?

4. If a boy walking at the rate of 4 miles an hour take 15 minutes to get to school, how long will he be if he walks at 3 miles an hour?

5. If 5 men take 12 days to do a certain work, how long will it take 3 men to do the same work?

6. If a cheese weighing 15 pounds is sold for 90 cents, what should be the cost of one weighing 24 pounds?

7. If 3 pounds of maple sugar be sold for 60 cents, how many pounds can be bought for a dollar ?

8. If a stove which burns 24 cents worth of coal a day consumes $36 worth in a winter, what will be the value of the coal required for the winter by one which uses 30 cents' worth a day ?

9. If, when flour is $6 a barrel, the 4-pound loaf costs 15 cents, what should the 4-pound loaf cost when flour is $8 a barrel ?

10. If *A* can run 10 yards while *B* runs 8 yards, what start can *A* afford to give *B* in a 100 yards dash ?

In 10 yards *A* gains 2 yards,
∴ “ 1 yard “ “ $\frac{2}{10}$ “
∴ “ 100 yards “ “ $\frac{2 \times 100}{10}$ or 20 yards.

11. If *A* can run 15 yards while *B* runs 12 yards, how much ahead will *A* be when he has run 100 yards ?

12. *A* runs 11 yards while *B* runs 9 yards ; what start can *A* afford *B* in a race of 220 yards ?

13. A father is four times as old as his son, and the sum of their ages is 60 years. How old is each ?

14. Two pounds of tea cost as much as three pounds of coffee, and five pounds of coffee cost $1.50. How much a pound is tea.

15. $2.40 will buy 3 bushels of barley or 5 bushels of oats, how much will 5 bushels of barley and 3 bushels of oats cost ?

16. A person bought two horses and a wagon for $210, and he paid for each horse twice as much as for the waggon. How much did he pay for each horse ?

17. A and B started together from the same place, going in the same direction. A travelled on horseback 51940 feet, while B on foot went one-fourth of the distance. How far were they apart?

18. A can run 11 yards whilst B can run 8 yards, by how many yards will A beat B in a course of 220 yards?

19. Sound travels at the rate of 1132 feet a second. How long would it take to travel 5660?

20. A is twice as old as B, and B is twice as old as C, and their united ages make forty-two years. What are their ages?

21. What number is contained 3169 times in 128991450, leaving a remainder of 474?

22. A father earns $5, whilst his son earns $3. When they have together earned $120, how much will belong to each?

23. John has $100; if he had $8 more he would have exactly six times as many as the half of what Henry has. How many has Henry?

24. A and B lived 990 yards apart. They both set out at the same time to walk to each others' houses and met in 6 minutes. A reached B's house in 9 minutes more. When did B get to A's house?

25. Divide $900 into two parts so that one may be $62 more then the other.

SECTION IV.

TABLES OF WEIGHTS & MEASURES.

MONEY TABLES.

57. Decimal Money. In this system the unit is divided into tenths (dimes) and hundredths (cents). In Canada and the United States the unit is of the same value and is called a dollar, \$1.

In France the unit is called a franc, and its value is nearly 20 cents, Canadian money. Several other countries of Europe have adopted a unit of the same value. Thus we have

\$1 = 10 dimes = 100 cents.
1 franc = 10 decimes = 100 centimes.

Note. The name dime is not used in Canada.

58. English Money. In this money the unit, called a pound, £1 ($\$4\cdot86\frac{2}{3}$), is divided into 20 shillings, the shilling into 12 pence, and the penny into 4 farthings. A farthing is denoted by $\frac{1}{4}$d. Thus we have

$\frac{2}{4}$d. or 2 farthings = 1 half-penny .... $\frac{1}{2}$d.
$\frac{4}{4}$d. or 4 farthings = 1 penny .... 1d.
12 pence = 1 shilling ...... 1s.
20 shillings = 1 pound .... £1.

The gold coins are the sovereign (a pound) and half sovereign. The silver coins are the crown (5s.), half crown, shilling and florin (2s. or £$\frac{1}{10}$), sixpence and threepence. The gold coin guinea (21s.) and silver coin groat (4d.) are out of use.

TABLES OF WEIGHTS.

59. Decimal or Metric. The unit of this weight is 1 gramme (15·32 grains Troy).

The sub-divisions are denoted by prefixes derived from the Latin. Thus Decigramme ($\frac{1}{10}$ gramme), Centigramme ($\frac{1}{100}$ gr.), Milligramme ($\frac{1}{1000}$ gr.). The multiples are denoted by prefixes derived from the Greek. Thus Dekagramme (10 grammes), Hectogramme (100 grammes), Kilogramme (1000 grammes). Thus we have

Milli.	Centi.	Deci.	Gramme.	Deka.	Hecto.	Kilo.
$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000

60. Troy Weight. This weight is used in weighing gold, silver and precious stones, and in scientific experiments. The unit, a pound, is divided into 12 ounces, an ounce is divided into 20 pennyweights, and a pennyweight into 24 grains. Thus we have

24 grains ...... = 1 pennyweight.. 1 dwt.
20 pennyweights = 1 ounce........ 1 oz.
12 ounces .... = 1 pound ...... 1 ℔.

Note. For medicinal purposes the ounce Troy is divided into 8 drachms, the drachm into 3 scruples, and the scruple into 20 grains. Thus we have

20 grains = 1 scruple .... ℈.
3 scruples = 1 drachm.......... ʒ.
8 drachms = 1 ounce .... ℥.
12 ounces = 1 pound .... 1 ℔.

APOTHECARIES' FLUID MEASURE.

61. In this measure, founded on the fact that a pint of pure water weighs 20 ounces :

60 minims......make 1 fluid drachm..fl. dr.
8 fluid drachms .... 1 fluid ounce...fl. oz.
20 fluid ounces ...... 1 pint..........o; octavius.
8 pints .... 1 gallon........c; congius.

62. Avoirdupois Weight. In this, which is the ordinary commercial weight of Great Britain and her Colonies, and of the United States, the unit, also called a pound, is divided into 16 ounces, and the ounce is divided into 16 drams. The multiples are quarter (of a hundredweight), hundredweight, and ton (2000 pounds). Thus we have

16 drams .... = 1 ounce .... 1 oz.
16 ounces ...... = 1 pound .... 1 ℔.
25 pounds ...... = 1 quarter .... 1 qr.
4 quarters ...... = 1 hundredweight .. 1 cwt.
20 hundredweights = 1 ton.

Note 1. By the Table for Troy weight, 1 oz. Troy = 480 grains, and 1 ℔. Troy = 5760 grs.; but by Act of Parliament, 1 ℔. Avoirdupois = 7000 grs. Troy, and therefore 1 oz Avoirdupois = $\frac{1}{16}$ of 7000 = $437\frac{1}{2}$ grs.

Note 2. An old method called long weight still used, especially in Great Britain, is to make the quarter 28 pounds. Thus we have

28 pounds ...... = 1 quarter .... 1 qr.
4 quarters ...... = 1 hundredweight (112 ℔s.) 1 cwt.
20 hundredweights = 1 ton (2240 ℔s.).

TABLES OF LINEAR MEASURE.

In measuring distances, lengths, breadths, heights, depths, etc., various units are used.

63. Decimal or Metric. In this system the unit is called a metre, and is about 39·37 inches long. It is subdivided and multiplied decimally, the same prefixes being used as in weight. Thus we have

Milli.	Centi.	Deci.	Metre.	Deka.	Hecto.	Kilo.
$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000

64. Common English Measure. In this the unit is 1 foot, and is divided into 12 inches. The inch is variously divided into eighths, sixteenths, tenths, etc. The multiples are given in the following table :

12 inches = 1 foot .... 1 ft.
3 feet .. = 1 yard .... 1 yd.
$5\frac{1}{2}$ yards .. = 1 rod, pole or perch.... 1 p.
40 rods .. = 1 furlong .... 1 f.
8 furlongs = 1 mile................. 1 m.
3 miles .. = 1 league .... 1 lea.

65. Chain Measure. In measuring land, the unit 1 foot is too small, so a chain, 4 rods long, called Gunter's chain, is made the unit, and is divided into 100 equal parts, called links. Thus

100 links = 1 chain = 4 rods = 22 yards.
1 furlong = 10 chains. 1 mile = 80 chains.

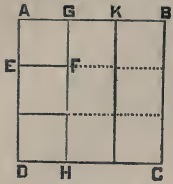
66. Cloth Measure. In this measure the yard is divided into quarters, and the quarter into 4 nails. Thus, a yard being 36 inches, 1 qr. = 9 in. 1 nail = $2\frac{1}{4}$ inches. Thus we have

$2\frac{1}{4}$ inches = 1 nail.
4 nails = 1 quarter.
3 quarters = 1 Flemish Ell.
4 quarters = 1 yard.
5 quarters = 1 English Ell.
6 quarters = 1 French Ell.

Note. The unit Hand (4 inches) is used in measuring the heights of horses, and the unit Fathom (6 feet) is used by sailors.

SQUARE MEASURE.

67. In this measure, which is used in computing the size of surfaces, such as land, floors, ceilings, walls, everything, in fact, in which length and breadth are to be taken into account, the various units are the squares of the corresponding units of length.



Suppose AB to represent a yard, divided into 3 feet, AG, GK, KB.

Now let AD, BC be drawn perpendicular to AB and equal to it, then the figure $ABCD$ represents a square yard. If each of the sides be divided into 3 equal parts, and lines be drawn as in the figure, $AEFG$ will represent a square foot. A square foot taken 3 times makes up $ADHG$, and $ADHG$ taken 3 times makes up the whole square. So that the square

= 1 square foot × 3 × 3.

= 1 square foot taken 9 times = 9 square feet.

Definition. The square of a number is the product of the number by itself.

We see then that the number of square feet in a square yard is the square of the number of feet in a linear yard or yard length.

In the same way, a square foot contains 1 sq. inch × 12 × 12, or 144 square inches.

68. Metrical Unit. In the metric system the unit is the square dekametre, containing 10 × 10, or 100 square metres, and it is called are. This is subdivided and multiplied decimally, so that we have

Milli.	Centi.	Deci.	Are.	Deka.	Hecta.	Kilo.
$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000

69. Common English System. In this system the unit is usually either a square foot or a square yard.

144 square inches = 1 square foot.
9 square feet . . = 1 square yard.
$30\frac{1}{4}$ square yards = 1 square pole or perch.
40 poles . . . . . . = 1 rood.
4 roods . . . . . . = 1 acre.
640 acres . . . . . . = 1 square mile.

70. Chain Measure. Since the linear chain is 4 rods, the square chain will contain 1 square rod × 4 × 4, or 16 square rods.

An acre .... = 40 rods × 4 = 160 rods.

∴ 1 acre ...... = 10 square chains.

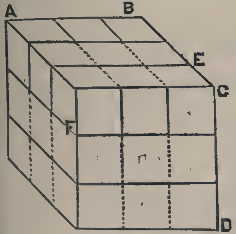
1 square chain = 100 × 100 or 10,000 square links.

1 acre ...... = 100,000 square links.

CUBIC MEASURE.

71. This measure is used to compute the bulk of stone, wood, earth, and everything in which length, breadth and depth are to be taken into account.

In this measure the various units are the cubes of the linear units.



Suppose the figure *AD* to represent a cubic yard, *i.e.* a figure which has six sides, each of which is a square yard. Each edge will be a linear yard, or 3 feet, and if the cube be divided as in the figure, then *FH* will represent a cubic foot. 1 foot taken 3 times makes up the block *FE*; the block *FE* taken 3 times makes up the slice *FB*; and the slice *FB* taken 3 times makes up the whole cube.

Thus 1 cubic yard .... = 1 cubic foot × 3 × 3 × 3.
= 27 cubic feet.

In like manner 1 cubic foot = 1 cubic inch × 12 × 12 × 12.
= 1728 cubic inches.

Note. 3 × 3 × 3 is called the cube of 3, and 12 × 12 × 12 is the cube of 12; and similarly with any other number.

Thus it will be seen that the number of cubic feet in a cubic yard is the cube of the number of feet in a linear yard, and the number of cubic inches in a cubic foot is the cube of the number of inches in a linear foot.

Note. A pile of wood 8 feet long, 4 feet high and 4 feet wide, is called a cord.

1728 cubic inches = 1 cubic foot.
27 cubic feet = 1 cubic yard.
40 cubic feet = 1 ton.
128 cubic feet = 1 cord.

72. The metric unit is a cubic metre and is called stere. Thus we have

Milli,	Centi.	Deci.	Stere.	Deka.	Hecto.	Kilo.
$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000

MEASURE OF CAPACITY.

73. This measure is used for measuring liquids, seeds, fruit, etc.

4 gills = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon.
2 gallons = 1 peck.
4 pecks = 1 bushel.
$2\frac{1}{2}$ bushels = 1 barrel (of flour).

Note. The last 3 measures are not used for liquids.

74. The **Metric Unit** is a cubic decimetre and is called litre. Thus we have

Milli.	Centi.	Deci.	Litre.	Deka.	Hecto.	Kilo.
$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000

TABLE OF GRAINS AND SEEDS.

75. The following table shews the weight of a bushel of articles named :

Wheat........	60 ℔s.	Flax seed	58 ℔s.
Potatoes	60 "	Corn	56 "
Onions	60 "	Rye..........	56 "
Peas	60 "	Buckwheat ..	48 "
Beans	60 "	Barley	48 "
Clover seed ..	60 "	Timothy seed .	48 "

And 34 ℔s. a bushel of Oats.

Note. By the "Weights and Measures" Act of 1873, the following articles may be sold by the cental of 100 ℔s. : Barley, beans, charcoal, corn, oats, peas, potatoes, rye, salt, seeds, and wheat. By the same Act, the Imperial bushel, containing eight Imperial gallons of 277·274 cubic inches in each, is the standard bushel in Canada. In Great Britain, 8 bushels make 1 quarter.

MEASURES OF TIME.

TABLE OF TIME.

76. 1 Second is written thus, 1″.

60 seconds...... make	1 minute..............	1′.
60 minutes	1 hour................	1 hr.
24 hours	1 day	1 day.
7 days..............	1 week	1 wk.
4 weeks, or 28 days ..	1 lunar month	1 mo.
365 days..............	1 civil or common year..	1 yr.

A year is divided into 12 months, called Calendar Months, the number of days in each of which may be easily remembered by means of the following lines :

Thirty days hath September,
April, June and November :
February has twenty-eight alone,
And all the rest have thirty-one :
But leap-year coming once in four,
February then has one day more.

Note. A civil or common year = 52 weeks, 1 day. A leap year = 366 days.

As the length of the year is nearly $365\frac{1}{4}$ days, a day is usually added every four years, viz,, when the number of the year is divisible by 4 ; but since this adds rather too much, the additional day is omitted 3 times in 400 years, viz., at the end of centuries whose numbers are not divisible by 4. Thus 1700, 1800, 1900, are not leap years, but 1600 and 2000 are. This makes the average length of the year $365\frac{97}{400}$, which is much more nearly correct than $365\frac{1}{4}$ days. The true length of the year is 365d. 5h. 48′ 48″. The addition of 1 day every 4 years is called the Julian correction ; the omitting of the addition 3 times in 400 years is called the Gregorian correction ; named after Julius Cæsar and Pope Gregory XIII., by whose authority the corrections were severally made.

MISCELLANEOUS TABLES.

77.

12 units ......make	1 dozen.
12 dozen	1 gross.
20 units	1 score.
24 sheets of paper ..	1 quire.
20 quires	1 ream.
14 pounds	1 stone.
14 stones or 196 lbs...	1 barrel of flour.
100 pounds	1 quintal.
200 pounds	1 barrel of pork or beef.

Note. When a sheet of printing paper is folded into 2 leaves, the size of the leaf is called folio ; into 4 leaves. quarto or 4to. ; into 8 leaves, octavo or 8vo. ; into 12 leaves, duodecimo, 12mo., etc.

REDUCTION.

Example 1. How many inches in 10 yds. 2 qrs. 3 nls. 1 in. ?

10 yds. 2 qrs. 3 nls. 1 in. 4	
42 qrs. 4	1 yd. = 4 qrs. ; ∴ 10 yds. 2 qrs. = (10 × 4) qrs. + 2 qrs. = 42 qrs.
171 nls. $2\frac{1}{4}$	1 qr. = 4 nls. ; ∴ 42 qrs. 3 nls. = (42 × 4) nls. + 3 nls. = 171 nls.
343 $42\frac{3}{4}$	1 nl. = $2\frac{1}{4}$ in. ; ∴ 171 nls. 1 in. = (171 × $2\frac{1}{4}$) in. + 1 in. = $385\frac{3}{4}$ in.
$385\frac{3}{4}$ in. Ans.	

Example 2. In 60812 grs. how many lbs., &c., Troy ?

24 { 4	60812 grs.	24 grs. = dwt ; ∴ 60812 grs. ÷ 24 = 2533 dwts. + 20 grs.
6	15203	
2,0	253,3 dwts. 20 grs.	20 dwts. = 1 oz., ∴ 2533 dwts. ÷ 20 = 126 oz. + 13 dwts.
12	126 oz. 13 dwts.	12 oz. = 1 lb., ∴ 126 oz. ÷ 12 = 10 lbs. + 6 oz.
	10 lbs. 6 oz.	

Therefore the result is 10 lbs. 6 oz. 13 dwts. 20 grs.

The process employed in Example 1, is called Reduction Descending, and that in Example 2, Reduction Ascending ; the one is the inverse of the other. If, therefore, we perform the one process on a given quantity, and on the result perform the other, we ought to get the original quantity—that is, we prove the correctness of our answer.

Note. When a quantity is expressed in one denomination only, it is called a simple quantity : as $6, 5 miles, 15 days.

When a quantity is expressed in several denominations, it is called a compound quantity: as 3 days, 5 hours, 18 minutes, £6 18*s.* 5¼*d.*

Example 3. Reduce £5, 19*s.*, 8½*d.* to half-pence.

$$\begin{array}{lll} \pounds. & s. & d. \\ 5 & 19 & 8\frac{1}{2} \\ 20 & & \\ \hline 119s. & & \\ \;\;12 & & \\ \hline \;\;1436d. & & \\ \;\;\;\;\;\;\;\;2 & & \\ \hline \;\;2873\text{ half-pence.} & & \end{array}$$

Hence,

78. For Reduction Descending:

Rule. Multiply the number of the highest denomination by the number of units of the next lower denomination in one unit of the highest, and to the product add the number of the lower denomination (if any). Continue the process till the required denomination is found.

Example 4. How many tons, &c., in 86436 oz. ?

$$\begin{array}{rr|ll} & 4 & 86436 & \text{oz.} \\ \cline{3-3} 16 & 4 & 21609 & \\ \cline{3-3} & 5 & 5402 & 4\text{ oz.} \\ \cline{3-3} 25 & 5 & 1080 & 2\text{ lbs.} \\ \cline{3-3} & 4 & 216 & \\ \cline{3-3} & 20 & 54 & \\ \cline{3-3} \end{array}$$

2 tons, 14 cwt., 2 lbs., 4 oz.

Hence,

79. For Reduction Ascending:

Rule. Divide the given number by the number of units which connect that denomination with the next higher, and set down the remainder as of the same denomination as its dividend. Carry on the process till you arrive at the denomination required.

EXERCISE XXVII.

1. Reduce 29 mls. 5 fur. 36 p. 4 yds. to feet.

2. How many ounces in 19 tons 1 qr. 5 lbs. 7 oz.?

3. Reduce 17 ac. 3 r. 13 p. 20 yds. to feet.

4. How many minutes in 12 days 17 hr. 45′?

5. How many £. *s.* *d.* in 60850 pence?

6. Reduce 10 yrs. 49 wks. 6 days 11 hrs. 30 sec. to minutes.

7. Reduce 907163 drams &c. to cwts.

8. How many pints in 136 bushels 2 qts.?

9. Reduce 909060 lbs. of wheat to bushels.

10. Reduce 103 cub. yds. 3 c. ft. to cubic in.

11. Reduce 1000000 lbs. of oats to bushels.

12. How many grains in 93 lbs. 8 oz. 13 dwts.?

13. In 10010010 sq. in. how many acres, &c.?

14. Reduce 11 lbs. 15 dwts. 3 grains to grains.

15. In 120878 lbs. of clover seed how many bushels?

16. Reduce 7 lbs. 7 oz. 7 drs. 1 sc. to grains.

17. How many miles &c. in 56812 inches?

18. Cotopaxi is 18875 feet high, and Chimboraro is 21424 feet. Express the difference in miles, &c.

19. How many seconds in a year containing 365 days 5 hrs. 48 min. 49 sec. ?

◆

COMPOUND ADDITION.

To find the sum of 2 mls., 3 fur., 5 per., 2 yds., 1 ft. ; 5 mls., 5 fur., 7 per., 4 yds., 2 ft., and 3 mls., 7 fur., 8 per., 5 yds., 2 ft.

mls.	fur.	per.	yds.	ft.
2	3	5	2	1
5	5	7	4	2
3	7	8	5	2
11	7	22	1	2

Writing the quantities so that numbers of the same name may be in the same vertical column, and adding the numbers in each column, beginning with the lowest, we find the sum of the numbers of feet to be 5 ft., *i.e.* 1 yard and 2 ft. ; put down the 2 ft. under the feet and carry the 1 yard to the yards ; then the sum of the yards is 12 yards or 24 half yards, which, divided by $5\frac{1}{2}$ or 11 halves, gives 2 per. and 2 half yards, *i.e.* 1 yard over. We write the 1 yard under the yards and carry the 2 per. to the column of perches. Proceeding in the same way with the remaining columns, we find the required result.

Hence,

80. To Add Compound Quantities:

Rule. Write the quantities, so that numbers of the same denomination may be in the same column, and draw a line below them.

Add the numbers of the lowest denomination together, and find how many units of the next higher denomination are contained in this sum.

Write the remainder, if any, under the column just added, and carry the quotient to the next column. Proceed thus with all the columns.

Note. The method of proof in the Compound Rules is the same as in the Simple Rules.

EXERCISE XXVIII.

Add together,

1.	$110.69
	203.25
	634.31
	863.67

	lbs.	oz.	dwt.	grs.
2.	14	8	15	5
	20	3	12	15
	22	9	14	16

	mls.	fur.	po.	yds,	ft.
3.	10	4	20	3	1
	37	1	25	3	2
	25	5	4	4	2

	£	s.	d.
4.	123	12	8½
	27	5	6
	9	15	9¾
	118	16	7

	lbs.	oz.	drs.	scs.	grs.
5.	28	2	7	2	19
	29	5	3	0	5
	8	3	6	1	12
	22	7	2	0	19

6.	$867.22
	768.27
	918.37
	1193.87

	tons.	cwt.	qrs.	lbs.	oz.	drs.
7.	225	13	2	5	8	6
	28	12	3	18	12	5
	201	16	1	15	3	8
	25	9	0	7	9	13
	206	19	2	17	11	15

	yds.	qrs.	nls.
8.	203	0	2
	209	3	3
	10	2	1
	506	1	0
	39	2	3

	wks.	dys.	hrs.	min.
9.	2	6	12	50
	3	2	15	25
	7	4	3	40
	8	1	6	35
	4	5	17	15

	c. yds.	ft.	in.
10.	18	18	127
	612	20	878
	207	5	909
	888	10	88
	38	12	809

	ac.	ro.	po.	yds.	ft.	in.
11.	809	2	20	16	8	140
	578	3	29	25	6	113
	127	1	21	30	5	107
	281	3	22	29	2	8
	39	2	37	18	8	119

12.	$1012.25
	973.08
	889.27
	83.67
	29.33

	lbs.	oz.	drs.	scs.	grs.
13.	8	8	6	2	5
	23	9	4	1	15
	11	10	5	2	16
	12	4	3	0	18
	28	7	7	0	19

	bu.	pk.	gal.
14.	60	2	1
	25	3	0
	12	0	1
	35	1	0
	28	3	1

15.	$8675.20
	907.18
	60.23
	875.09
	28.37

◆

COMPOUND SUBTRACTION.

Example. From 2 ac., 3 ro., 16 per., 8 yds., 2 ft., take 1 ac., 2 ro., 25 per., 20 yds., 6 ft.

ac.	ro.	per.	yds.	ft.	
2	3	16	8	2	
1	2	25	20	6	
1	0	30	17	5	
				2	36
1	0	30	17	7	36 in.

The less quantity is written underneath the greater, so that numbers of the same name are under each other.

I cannot take 6 ft. from 2 ft., so I add 1 yard or 9 ft. to the 2 ft., making it 11 ft.; then 6 from 11 leaves 5; write down the 5 under the ft.; in order to increase the lower number equally with the upper, I add 1 yd. to 20 yds., making it 21 yds.; then, since 21 yds. cannot be taken from 8 yds., I add 1 per. or $30\frac{1}{4}$ yds. to the 8 yds., making it $38\frac{1}{4}$ yds.; then 21 yds. from $38\frac{1}{4}$ yds. leaves $17\frac{1}{4}$ yds.;

I write down the 17 yds., and for the quarter sq yd. write down 2 ft. 36 in. for subsequent addition. I work the remaining columns in the same way, and find the required answer.

Hence,

81. To Subtract Compound Quantities:

Rule. Place the less quantity under the greater, so that numbers of the same name may be under each other in the same column, and draw a line below them.

Begin at the right, and subtract, if possible, each number of the lower line from that which stands above it, and set the remainder underneath.

But when any number in the lower line is greater than the number above it, add to the upper one as many units as make one unit of the next higher denomination; subtract as before, and carry one to the number of the next higher denomination in the lower line.

Proceed thus throughout all the columns.

EXERCISE XXIX.

1.	$678.33
	593.43

	£	*s.*	*d.*
2.	97	13	6¼
	83	15	8

	tons.	cwt.	qrs.	lbs.	oz.
3.	315	2	3	17	8
	298	17	0	20	15

	yrs.	wks.	dys.	hrs.	min.
4.	227	40	5	12	5
	219	45	6	50	55

	lbs.	oz.	drs.	scs.	grs.
5.	29	10	4	0	15
	24	11	7	2	18

	bu.	pk.	gal.
6.	917	2	2
	889	3	3

	lbs.	oz.	dwts.	grs.
7.	10	10	10	20
	8	11	19	23

	mls.	fur.	po.	yds.	ft.	in.
8.	99	6	26	4	2	6
	89	7	33	2	1	9

	c. yds.	ft.	in.
9.	1000	20	1000
	997	26	1589

	yds.	qrs.	nls.
10.	208	2	1
	199	3	3

	ac.	ro.	po.	yds.	ft.	in.
11.	28	1	17	30	8	50
	18	3	37	25	7	117

	tons.	cwt.	lbs.	oz.	drs.
12.	201	5	8	5	8
	200	17	22	11	14

13.	$878.06
	689.38

◆

COMPOUND MULTIPLICATION.

Example 1. Multiply £21 6s. $8\frac{1}{2}$d. by 256.

$$256 = 200 + 50 + 6$$
$$= 10 \times 10 \times 2 + 10 \times 5 + 6.$$

£	s.	d.	
21	6	$8\frac{1}{2}$	
		10	
213	7	1	(10 times)
		10	
2138	10	10	(100 times)
		2	
4267	1	8	(200 times)
1066	15	5	(50 times)
128	0	3	(6 times)
5461	17	4	(256 times).

The above neat method is frequently used for questions of this kind, but the following is the shortest and on the whole the simplest :

	cwt.	gr.	lbs.	oz.
Example 2.	2	1	5	6
				256
	589	3	1	0

16)1536
96 0 oz.
1280
25)1376
55 1 lb.
256
4)311
77 3 grs.
512
589 cwt.

We multiply 6 oz. by 256, giving 1536 oz. or 96 lbs. and no oz. Set down 0 oz. and carry 96 lbs. to the lbs. ; then multiply 5 lbs. by 256 and add the 96, giving 1376 lbs. or 55 grs. and 1 lb. ; write down 1 lb. and carry 55 grs. to the grs. ; then multiply 1 gr. by 256 and add the 55, giving 311 grs. or 77 cwt. and 3 grs. ; write down 3 grs. and carry 77 cwt. to the cwts. ; then multiply 2 cwt. by 256 and add the 77 cwt., giving 589 cwt. Thus the product is 589 cwt., 3 grs., 1 lb.

Hence,

82. To Multiply Compound Quantities:

Rule. Write the Multiplier under the lowest denomination of the Multiplicand, and draw a line below. Beginning at the right, multiply each of the columns in succession by the Multiplier, and reduce, write down, and carry exactly as in Compound Addition.

EXERCISE XXX.

	£	s.	d.
1.	18	12	8½
			10

	lbs.	oz.	dwt.	grs.
2.	8	5	6	16
				9

3. $212.65
15

	cwt.	qrs.	lbs.	oz.
4.	6	3	5	14
				12

	mls.	fur.	per.	yds.	ft.
5.	2	5	8	2	2
					27

	lbs.	oz.	drs.	grs.
6.	5	6	6	15
				18

	dys.	hrs.	min.
7.	8	16	26
			17

	ac.	ro.	per.	yds.
8.	25	2	23	3
				43

	yds.	qrs.	nls.	in.
9.	12	2	3	2
				55

	bu.	pk.	qt.
10.	102	3	3
			117

	dys.	hrs.	min.	sec.
11.	365	5	48	57
				223

	c. yds.	ft.	in.
12.	1008	21	112
			654

	yrs.	wks.	dys.	min.
13.	100	42	6	30
				512

14. How much ditching will a man do in nine weeks of six days each, if he can do 3 poles 2 yards 1 foot daily?

15. How much cloth will be required to make 6 coats, 2 yds. 1 qr. each, 7 waistcoats at 3 qrs., and 16 pairs of trousers requiring 1 yd. 3 qrs. 1 nl. each?

16. A cubic foot of water weighs 62 lbs. 7 oz. 4 drs. What is the weight of 236 cubic feet?

17. What is the amount of the following grocer's bill: 15 lbs. of sugar at 9 cents, 5 lbs. of tea at 75 cents, 7 lbs. of candles at 20 cents, 5 gals. of coal oil at 41 cents, and 16 lbs. of soap at 12 cents?

18. If a man is to get $2.50 a day, how much should he receive for 289 days at the same rate?

19. Sold 3 pieces, each 5 yds. 1 qr. 1 nl., from a piece of alpaca measuring 165 yards. What is the length of the remnant?

20. How many cwt., &c., are in equal in weight to 18 times 107 lbs. 6 oz. 16 dwt. 15 grs.?

COMPOUND DIVISION.

Example 1. Divide 2 tons 18 cwt. 1 qr. 5 lbs. 8 oz. by 23.

```
   tons. cwt.  qr.  lbs.  oz. 0 tons.
23)2     18    1    5     8(2   2
   20 [add the 18 cwt.]
   —
23)58(2 cwt.
   46
   —
   12
    4 [add the 1 qr.]
   —
23)49(2 qr.
   46
   —
    3
   25 [add the 5 lbs.]
   —
23)80(3 lbs.
   69
   —
   11
   16 [add the 8 oz.]
   ——
23)184(8 oz.
```

23 will not divide 2 tons; but 2 tons 18 cwt. is 58 cwt. 58 cwt. divided by 23 gives 2 cwt. and 12 cwt. over; 12 cwt. 1 qr. is 49 qrs. 49 qrs. divided by 23 gives 2 qrs. and 3 qrs. over; 3 qrs. 5 lbs. is 80 lbs. 80 lbs. divided by 23 gives 3 lbs. and 11 lbs. over; 11 lbs. 8 oz. is 184 oz. 184 oz. divided by 23 gives 8 oz.

∴ the quotient is 2 cwt. 2 qrs. 3 lbs. 8 oz.

The above is an example of a concrete quantity divided by an abstract one, giving a concrete quotient of the same kind as the dividend.

Hence,

83. To Divide Compound Quantities:

Rule. Place the quantities as in Simple Division; then find how often the divisor is contained in the highest denomination of the dividend; put this number down in the quotient; multiply as in Simple Division and subtract.

If there be a remainder, reduce it to the next lower denomination, adding in the number of that denomination in the dividend, and repeat the division. Carry on this process through the whole of the dividend.

Example 2. How many times does 4 weeks 5 days 6 hours contain 6 days 4 hours 12 minutes ?

dys. hrs. min.	wks. dys. hrs.	
6 4 12	4 5 6	8892)47880(5
24	7	44460
148	33	3420
60	24	
8892 min.	798	*i. e.* 5 times with remainder 3420 min.
	60	
	47880 min.	

This is an example of dividing one concrete quantity by another, the quotient being abstract *i.e.*, a mere number, number of times. In doing this question and similar ones, we reduce each quantity to the lowest denomination given in either, and divide as in Simple Division.

EXERCISE XXXI.

1. Divide £6565. 4*s.* 8*d.* by 44.

2. Divide 198 ac. 3 ro. 16 per. 3 yds. 72 in. by 187.

3. Divide 3 tons 5 cwt. 2 qrs. by 56.

4. Divide 5 lbs. 8 oz. 6 dwt. 2 grs. by 55.

5. Divide 8 c. yds. 9 ft. by 121.

6. Divide $243.36 by 117.

7. How many times is 195 yards 1 ft. 8 in. contained in a mile ?

8. The business of a Company produces £24299. 14*s.* 5*d.* Of this amount, £14487. 16*s.* 11*d.* goes for working expenses, and the remainder is equally divided among 100 shareholders. How much does each get?

9. How many sovereigns are there in 80 lbs. of standard gold, an ounce of standard gold being worth £3. 17*s.* 10½*d.*?

10. If £4. 16*s.* buy 288 yards of cloth, how many can be bought for £1. 11*s.* 7½*d.*?

11. If a gallon contains 277·274 cubic inches, how many cubic yards are there in 100 bushels?

12. Thirty-three telegraph posts, placed at equal distances, extend a mile. How far apart are the posts?

◆

Miscellaneous Examples.

PAPER I.

1. How much coffee at 40 cents per lb. must be given in exchange for 78 lbs. of tea at 75 cents?

2. In 29561728 ounces (avoirdupois) how many tons, cwt., &c.?

3. Reduce 5 furlongs 97 yards 2 feet 3 inches, to inches.

4. Multiply £7365 9*s.* 4¼*d.* by 97.

5. In 1000000000 inches how many miles, &c.?

6. If 1000 sovereigns weigh 21 lbs. 5 oz. 16 dwts., what weight of gold will be contained in 192 sovereigns?

PAPER II.

1. How many rods are there in 10 miles 5 furlongs 7 chains 11 yards?

2. Make out the bill for the following articles: $10\frac{1}{4}$ yds. of ribbon, at 9 cents a yard; 13 yards 2 qrs. 3 nails of calico, at 16 cents a yard; 106 reels of cotton, at 16 cents a dozen; 10 pieces of tape, at 6 cents a piece; and one umbrella, at $2.25.

3. If 25 lbs. of coffee at 25 cents a lb. be mixed with 200 lbs. at 40 cents a lb., what is the value per pound of the mixture?

4. If 25 chests of tea weigh 1 ton 6 cwt. 1 qr., how many chests of the same size will weigh 1 ton 15 cwt. 2 qrs. 20 lbs.?

5. How many yards of cloth worth 18*s.* 3*d.* per yard, ought to be given in exchange for 24 English ells of cloth worth 13*s.* $8\frac{1}{4}$*d.* per ell?

6. A surveyor's chain is 22 yards long, and is divided into 100 links. How many links are there in $2\frac{1}{2}$ acres?

PAPER III.

1. How many days are there altogether in the months of February, March, April, May, June and July in leap year?

2. How many pounds are there in 91200 farthings?

3. An ounce of gold being worth £3 17*s.* $10\frac{1}{2}$*d.*, how many sovereigns are there in 80 lbs. of standard gold?

4. Multiply 17943 by 5079; and divide 348753392 by 688.

5. Write out the table of avoirdupois weight, and show that 144 lbs. avoirdupois = 175 lbs. troy.

6. Divide 2350 acres of land among 47 people, and find what each person's share is worth at $3.50 per acre.

PAPER IV.

1. The circumference of a waggon-wheel is 12 ft. 6 in. How often does it turn ronnd in running a league?

2. How many pounds avoirdupois are equivalent to 109375 pounds troy?

3. If the railway fare for a journey of 50 miles be $1.50, what will be the fare for a journey of 120 miles?

4. A gallon contains 277·274 cubic inches, how many cubic yards are there in 100 bushels?

5. If a schoolroom is 25 ft. long and 20 ft. wide, how many children will it accommodate, allowing for each of them 8 superficial ft. at the least? And if the room is 10 ft. 4 in. high, what cubical space is there for each child?

6. A dealer buys 15 horses at $140 each, and sells 8 of them at $185 each, and the remainder at $210.50 each: find his total gain.

PAPER V.

1. If 10 oz. 13 dwts. 16 grms. of gold worth £3 5*s.* per oz., be mixed with 1 oz. 2 dwts. 8 grs. of silver worth 5*s.* per oz., how much worthless alloy must be added to reduce the value of the compound metal to £2 16*s.* 8*d.* per oz.?

2. Divide 198 acres 3 roods 16 perches 3 yards 72 inches equally among 187 men.

3. A freight train is 8 miles ahead of an express, which travels at the rate of 2 miles in 3 minutes. It is run into in 24 minutes. How many miles an hour was the freight train running?

4. A farmer sells 250 bushels of wheat at $1.15 per bushel. He has to pay the price of 10 bushels of clover seed at $3.50 per bushel, of 5 sheep at $6 each, and of a

horse worth $150. How much is left of the price of the wheat?

5. The crop on a farm of an hundred acres is as follows: 32 acres of wheat, average yield 21 bushels to the acre, 8 acres of barley, 35 bushels to the acre, and 18 acres of oats, 45 bushels to the acre. Find how much the whole crop is worth, supposing wheat worth $1.25, barley 75 cents, and oats 36 cents per bushel.

6. Find the cost of 19 trees, each containing on an average 173 cubic feet of wood, at 22 cents the cubic foot.

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SECTION V.

FACTORS AND MULTIPLES.

84. A **factor** of any given nnmber is a number that will divide it exactly, *i. e.*, without any remainder.

Thus 4 is a factor of 8, because 4 is contained twice exactly in 8.

85. A **common factor** of two or more given numbers is a number which will divide each of the given numbers exactly.

Thus 5 is a common factor of 20, 35, and 45.

86. The **greatest common factor** (G. C. F.) of two or more given numbers is the greatest number which will divide eoch of the given numbers exactly.

Thus 5 is greatest common factor of 15, 20, and 25.

The greatest common factor is often called the greatest common measure (G. C. M.

87. To find the greatest common factor of any two numbers,

Rule. Divide the greater number by the *less*. If there be a remainder, divide the first divisor by it. If there still be a remainder, divide the second divisor by this remainder, and so on, always dividing the last preceding divisor by the last remainder, till nothing remains. The last divisor will be the greatest common factor required.

Ex. 2. Find the G. C. F. of 175 and 225.

```
175)225(1
    175
    ---
     50)175(3  bringing down last divisor, 75 for dividend.
        150
        ---
         25)50(2  "     "     "    50    "
            50
            —  ∴ 25 G. C. F. required.
```

88. To find the G. C. F. of three or more numbers, first find the G. C. F. of any two; then the G. C. F. of this G. C. F., and a third number and so on. The last G. C. F. will be the G. C. F. of all the numbers.

EXERCISE XXXII.

Find G. C. F. of :—

1. 544 and 736.
2. 6993 and 8991.
3. 3717 and 2065.
4. 6307 and 10812.
5. 5472 and 4256.
6. 2517 and 3356.
7. 2261 and 8303.
8. 2737 and 10057.
9. 12019 and 44137.
10. 3212 and 9052.
11. 313441 and 111221.
12. 3835 and 6939.
13. 484 and 308.
14. 5515 and 12133.
15. 15453 and 20907
16. 300300 and 8721.

17. Three drovers, A, B and C, bought sheep at the same rate per head; A's flock cost $102, B's $138, and C's $87; find how many each bought.

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89. Multiple. One number is a multiple of another when it can be divided by that other exactly.

Thus, 12 is a multiple of 3, because 3 divides 12 without remainder.

90. A common multiple of two or more numbers is a number that each of them will divide exactly.

Thus, 16 is a common multiple of 2 and 4, for 2 and 4 will each of them divide 16 without remainder.

91. The least common multiple (L. C. M.) of two or more numbers is the least number that can be divided by each of them exactly.

Thus, 12 is the least common multiple of 2, 3, 4 and 6.

92. To find the least common multiple of two or more numbers.

Find the L. C. M. of 18, 24, 36 and 42.

$$18 = 2 \times 3 \times 3.$$

$$24 = 2 \times 2 \times 2 \times 3.$$

$$36 = 2 \times 2 \times 3 \times 3.$$

$$42 = 2 \times 3 \times 7.$$

The least common multiple must contain as factors, $2 \times 3 \times 3$ for 18; $2 \times 2 \times 2 \times 3$ for 24; $2 \times 2 \times 3 \times 3$ for 36; $2 \times 3 \times 7$ for 42; therefore the L. C. M. must be $2 \times 2 \times 2 \times 3 \times 3 \times 7$ or 504; for it contains the factors of each of these numbers, and since it contains no others, it is the least common multiple.

A more convenient practical method of working examples like the above is the following :—

$$\begin{array}{r|rrrr} 2 & 18, & 24, & 36, & 42 \\ \hline 2 & 9, & 12, & 18, & 21 \\ \hline 3 & 9, & 6, & 9, & 21 \\ \hline 3 & 3, & 2, & 3, & 7 \\ \hline & 1, & 2, & 1, & 7 \end{array}$$

The L. C. M. is 2 × 2 × 3 × 3 × 2 × 7 or 504. By this method we readily find the factors of the numbers.

Note. The process of finding the L. C. M. may often be shortened by striking out in the same line every number which exactly divides any other number in that line.

Example 1. Find L. C. M. of 6, 8, 10, 12, 15.

$$\begin{array}{r|rrrrr} 2 & \cancel{6}, & 8, & 10, & 12, & 15 \\ \hline 2 & & 4, & \cancel{5}, & 6, & 15 \\ \hline & & 2, & & \cancel{3}, & 15 \end{array}$$

∴ L. C. M. is $2 \times 2 \times 2 \times 15$ or 120.

Explanation. Every multiple of 12 must be a multiple of 6; therefore strike out 6; for the same reason strike out 5 in second line and 3 in the third line.

Rule. Arrange the numbers in a line from left to right, with a comma placed between every two; divide, beginning with the LOWEST PRIMES, 2, 3, &c., those numbers which have a common factor by that common factor, and place the quotients and the undivided numbers in a line beneath and separated as before; proceed in the same way until a row of numbers is obtained in which there are no two numbers which have a common factor greater than unity. The continued product of all the divisors, and the numbers in the last line, will be the least common multiple.

EXERCISE XXXIII.

Find the L. C. M. of :—

1. 3, 5, 7.
2. 8, 16, 14.
3. 12, 15, 25.
4. 8, 12, 10, 6.
5. 11, 8, 6, 19.
6. 44, 121, 19, 76.
7. 15, 19, 60, 57.
8. 14, 21, 84, 72.
9. 5, 9, 11, 7, 21, 3.
10. 32, 3, 8, 6, 4.
11. 13, 38, 39, 19.
12. 12, 15, 18, 21.
13. 3, 6, 9, 12, 15.
14. 8, 16, 24, 15, 30.
15. 15, 65, 75, 85.
16. 72, 54, 108, 500.

17. The even numbers from 2 to 12 inclusive.

18. The odd numbers from 3 to 13 inclusive.

18. What is the least sum of money for which I can buy either copy-books at 10 cents each, or slates at 15 cents each, or arithmetics at 25 cents each, or fourth-readers at 35 cents each?

19. A battalion of soldiers numbering less then 1000 can be divided off into companies of 60, of 72, of 80, or of 90 men. Find the number in the battalion.

20. Find the least number which when divided by 12, 15, 18, 20, or 24, always leaves remainder 5.

21. In a certain length of fencing it is found that whether the posts be put 8, 9 or 10 feet apart, there is always a distance of 6 feet left over. How long must the fence be?

22. When a certain number of dozens of oranges are counted by fives or by sevens there are 3 over. Find how many there are.

Miscellaneous Examples.

PAPER I.

1. Three wheels, making 60, 36 and 24 revolutions in a minute, start with certain points in their circumference downwards. When will they first come all together again in the same position?

2. Subtract two hundred and forty-three thousand six hundred and one, from five million seven hundred thousand aud fifty, and multiply the remainder by twenty-three.

3. Find the greatest number of which 33495, 106260 are multiples; and the least number of which 26, 33, 39, 44 are divisors.

4. Divide two million four hundred and ninety thousand two hundred and one, by four thousand and eighty-nine.

5. Two cogged wheels work together, there being 32 cogs on one and 36 on the other. The larger wheel makes 36 revolutions in a second. How often will the same cogs be in contact in the same position?

6. Two velocipedes, A and B, start together along a road at the rate of 6 and 7 miles an hour respectively. The circumference of A's guiding-wheel is 40 in., that of B's 36 in. How often will the same points of the guiding-wheels which were in contact with the ground at starting, be simultaneously in contact with the ground during the first hour of their progress.

PAPER II.

1. When is one number said to be the G. C. F. of two others? Find G. C. F. of 1575 and 3885.

2. The annual divisible receipts of a railway company are $437000, and there are 250000 shares. How much would this give for each share? and how much would a person get who had ten shares?

3. In dividing one whole number by another, what does the quotient determine? Divide 243584 by 346.

4. Can the G. C. F. of two numbers exceed the difference between them?

5. In finding the G. C. F. of two numbers, how is it indicated that the numbers are prime to each other?

6. Find the least number which is divisible by all the numbers from 1 to 12.

PAPER III.

1. Multiply 280795 by 80693, and prove that your answer is correct by division.

2. The product of two numbers is 17037006, and one of them is 4858, what is the other?

3. Find the G. C. F. of 441441 and 844272, and the L. C. M. of 7, 11, 21, 63, 91, 99, 117, 143.

4. Find the sum of four hundred and three thousand and twelve, seven million twenty-four thousand and five, five hundred and one million five hundred and one thousand and eighteen.

5. What is the name and meaning of the following signs: $+$, $=$, $\times$, $\div$?

6. Find the difference between $10 + 624 \times 383 \div 32$ and 7000.

PAPER IV.

1. A farmer took to the market 25 bags of wheat; 5 of them weighed 126 lbs. each, 8 weighed 118 lbs. each, 10 weighed 132 lbs. each, and the remainder 135 lbs. each. The bags on an average weigh 2 lbs. each. Allowing 60 lbs. to the bushel, how many bushels had the farmer?

2. How many years in 438000 days (a year = 365 days)?

3. There are 100 bags of oats to be weighed: 15 of them weighed 98 lbs. each, 27 weighed 103 lbs. each, and 40 weighed 111 lbs. each, the remainder of them 87 lbs. each. Allowing 150 lbs. for the weight of all the bags, find how many bushels of oats were there? (a bushel = 34 lbs.)

4. A farmer sold 5 sheep at $6 each, 3 head of cattle at $40 a head, and a horse for $115. He had to take home 40 lbs. of sugar at 9 cents a lb., 10 lbs. of tea at 60 cents a lb., 36 yards cotton at 8 cents a yard, and 9 yards of tweed cloth at $1.15 a yard. Find how much he took home, allowing $1.50 for his expenses.

5. Find the sum of 875, 909, 1207, 8889, 70807, 55639, 83358, and from the sum take one hundred thousand nine hundred and seven.

6. If 3 ducks are worth 4 chickens, and 3 geese are worth 10 ducks, find the value of a goose; a pair of chickens being worth 60 cents.

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SECTION VI.

VULGAR FRACTIONS.

93. To explain the nature of a fraction, let us take the unit of money, one dollar. This is divided into four equal parts called quarters. One of these might be taken as unit (lower unit) and denoted by 1, then two quarters would be 2, three would be 3, four, or a whole dollar, would be 4, a dollar and a quarter would be 5, and so on. But in order to shew that the new unit or 1 is one of the four equal parts of another unit, 1 dollar, the figure 4 is written under the number which tells how many of the parts (lower units) are taken: thus 1 quarter is written $\frac{1}{4}$, 2 quarters $\frac{2}{4}$, 3 quarters $\frac{3}{4}$, 4 quarters $\frac{4}{4}$, 5 quarters $\frac{5}{4}$, and so on.

The dollar is also divided into ten equal parts, and then 1 of these is $\frac{1}{10}$ of a dollar, 2 parts are $\frac{2}{10}$, 3 parts $\frac{3}{10}$, and the whole dollar is $\frac{10}{10}$.

Such expressions as $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{4}$, etc., are called Vulgar (Common) Fractions.

94. A Fraction is an expression which denotes one or more of the equal parts of a unit.

95. Numerator. The upper number, which shews how many parts are taken, is called the Numerator.

96. Denominator. The lower number, which shews into how many parts the unit is divided, is called the Denominator.

97. Terms. The Numerator and Denominator of a fraction are called its Terms.

98. Fractions which have the same denominator can be treated like whole numbers : thus

$$\frac{3}{10} + \frac{2}{10} = \frac{5}{10}$$

The unit is divided into 10 equal parts and 3 parts + 2 parts = 5 parts :

$$\frac{7}{8} - \frac{5}{8} = \frac{2}{8}$$

The unit is divided into 8 equal parts and 7 parts − 5 parts = 2 parts :

$$\frac{5}{11} \times 2 = \frac{10}{11} \quad \left(\frac{5}{11} + \frac{5}{11} = \frac{10}{11}\right)$$

The unit is divided into eleven equal parts and 5 parts taken twice = 10 parts :

$$\frac{12}{13} \div 4 = \frac{3}{13}$$

The unit is divided into 13 equal parts and 12 parts ÷ 4 = 3 parts.

99. A Fraction may denote a Quotient.

To shew that $\frac{2}{3}$ may denote $2 \div 3$.

When the unit is divided into 3 equal parts, 2 units will contain 6 such parts :

$$\begin{aligned} \therefore\ 2 \div 3 &= 6 \text{ such parts} \div 3 \\ &= 2 \text{ such parts} \end{aligned}$$

But when the unit is divided into 3 equal parts, 2 such parts are denoted by $\frac{2}{3}$; $\therefore\ 2 \div 3 = \frac{2}{3}$

$$\text{or } \tfrac{1}{3} \text{ of } 2 = \tfrac{2}{3} \text{ of } 1$$

100 A Mixed Number is one made up of a whole number and a fraction : such as $1\frac{1}{4}$, $2\frac{3}{4}$.

101. A Proper Fraction is one whose numerator is less than its denominator.

An Improper Fraction is one whose numerator is not less than its denominator. Thus $\frac{3}{4}$, $\frac{5}{6}$ are Proper Fractions, and $\frac{5}{4}$, $\frac{7}{6}$ are Improper Fractions.

102. We can suppose a part of a whole to be sub-divided.

For example, half a cake, $\frac{1}{2}$, may be sub-divided into 3 equal parts ; then 1 of these parts will be $\frac{1}{3}$ of $\frac{1}{2}$, 2 parts will be $\frac{2}{3}$ of $\frac{1}{2}$.

Such expressions are called Compound Fractions.

A Simple Fraction is one in which the numerator and denominator are both whole numbers ; and it is a fraction of 1 : as $\frac{3}{4}$, $\frac{5}{3}$.

A Compound Fraction is a fraction of any other number than 1 : as $\frac{2}{3}$ of $\frac{5}{6}$, $\frac{3}{5}$ of $4\frac{1}{2}$.

103. Complex Fractions are such expressions as $\frac{\frac{2}{3}}{5}$, $\frac{4}{\frac{6}{7}}$, $\frac{\frac{3}{4}}{\frac{8}{9}}$, $\frac{4\frac{1}{2}}{5\frac{2}{3}}$, in which the numerator and denominator are not both whole numbers ; but such expressions ought rather to be considered as quotients : thus $\frac{2}{3} \div 5$ $4 \div \frac{6}{7}$, $\frac{3}{4} \div \frac{8}{9}$, $4\frac{1}{2} \div 5\frac{2}{3}$.

104. Since $1 = \frac{4}{4}$, $\therefore\ 2 = \frac{8}{4}$ and $3 = \frac{12}{4}$, $2\frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$, $3\frac{2}{5} = \frac{15}{5} + \frac{2}{5} = \frac{17}{5}$.

So $\frac{13}{5} = \frac{10}{5} + \frac{3}{5} = 2\frac{3}{5}$, $\frac{29}{4} = \frac{28}{4} + \frac{1}{4} = 7\frac{1}{4}$.

Hence we have the following Rules :

I. To express a whole number as a fraction with any given denominator.

Rule. Multiply the whole number by the given denominator, and under the result, as numerator, write the given denominator.

EXAMPLE. Express 5 as a fraction with denominator 3.

$$5 = \frac{15}{3}$$

II. To convert a mixed number into a simple fraction.

Rule. Multiply the whole number by the denominator, and add the numerator. The result will be the numerator of the required fraction, and the given denominator will be the denominator.

EXAMPLE. Express $17\frac{3}{5}$ as a simple fraction.

$$17\frac{3}{5} = \frac{85}{5} + \frac{3}{5} = \frac{88}{5}$$

III. To reduce an improper fraction to a whole or mixed number.

Rule. Divide the numerator by the denominator. If there be no remainder the result will, of course, be a whole number. If there be a remainder it will be the numerator of the fractional part, and the given denominator will be the denominator.

EXAMPLES. $\frac{18}{3} = 6$, $\frac{23}{5} = \frac{20}{5} + \frac{3}{5} = 4\frac{3}{5}$.

Note. Since $\frac{23}{5}$ also means $23 \div 5$, instead of saying the quotient is 4 with remainder 3, we can now say the complete quotient is $4\frac{3}{5}$.

EXERCISE XXXIV.

1. If a unit be divided into five equal parts, express one, two, three, four, and five parts, respectively, as fractions.

2. If you divide a cake into eight equal parts and give away five parts, what fractions will denote, respectively, how much is given away, and how much is left?

3. Express 7 quarters as a fraction of a dollar; also as a mixed number.

4. How much more than 1 is $\frac{7}{5}$?

5. How much less than 1 is $\frac{5}{8}$?

6. Which is greater, $\frac{3}{4}$ or $\frac{5}{6}$?

7. If a quarter of a dollar be divided into 5 equal parts, express 2 of these as a compound fraction of a dollar.

8. Express as simple fractions $1\frac{3}{5}$, $2\frac{2}{5}$, $3\frac{5}{6}$, $4\frac{1}{8}$, $5\frac{3}{8}$, $13\frac{8}{9}$, and $6\frac{4}{17}$.

9. Convert to mixed numbers $\frac{4}{3}$, $\frac{6}{5}$, $\frac{11}{8}$, $\frac{25}{7}$, $\frac{29}{12}$, $\frac{35}{16}$, $\frac{49}{15}$, and $\frac{100}{19}$.

10. Express as whole numbers $\frac{6}{6}$, $\frac{12}{6}$, $\frac{18}{6}$, $\frac{28}{14}$, $\frac{100}{25}$, $\frac{8}{1}$, $\frac{96}{16}$, and $\frac{105}{15}$.

11. Express 1, 3, 5, 8, 13 and 20, respectively, as fractions with denominator 4.

12. Express 13, 14, 15, 16, 20 and 25, respectively, as fractions with denominator 5

13. If a pound be divided into 16 equal parts called ounces, what fractions of a pound will five ounces, a pound

and a half, and two pounds and three-quarters, respectively be?

14. If four pounds and a half be divided into three equal parts, how will two of these be denoted?

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105. When the unit of money, a dollar, is divided into 2 equal parts (halves), if the dollar be denoted by 1, each half will be denoted by $\frac{1}{2}$.

When the dollar is divided into 4 equal parts (quarters), each quarter will be denoted by $\frac{1}{4}$, and since a half is now divided into 2 quarters, we see that $\frac{1}{2} = \frac{2}{4}$.

Again, when the dollar is divided into 10 equal parts, each part will be $\frac{1}{10}$, and since a half dollar is divided into 5 such parts, we see that $\frac{1}{2} = \frac{5}{10}$.

So when the dollar is divided into 20 equal parts, or into 100 equal parts (cents), we will have $\frac{1}{2} = \frac{10}{20} = \frac{50}{100}$, $\frac{1}{4} = \frac{5}{20} = \frac{25}{100}$, $\frac{3}{4} = \frac{15}{20} = \frac{75}{100}$.

Since $\frac{1}{2}$, $\frac{2}{4}$, $\frac{5}{10}$, $\frac{10}{20}$, $\frac{50}{100}$, all denote the same value, they are called equivalent fractions.

The pupil will now see the truth of the following important principle:

The value of a fraction is not altered by multiplying, or by dividing both terms by the same number.

Note. A fraction also denotes a quotient, and it is equally true that the value of a quotient is not altered by multiplying, or by dividing both dividend and divisor by the same number.

106. Lowest Terms. When the numerator and denominator are both whole numbers and have no common factor, the fraction is said to be in lowest terms.

107. To reduce a Fraction to its lowest terms.

Rule. Divide the numerator and denominator by their greatest common factor (G. C. F).

EXAMPLE 1. Reduce $\frac{36}{48}$ to its lowest terms.

The G. C. F. of 36 and 48 is 12. Dividing both terms of the fraction by 12 (which does not alter its value), we have 12)$\frac{36}{48} = \frac{3}{4}$.

EXAMPLE 2. Reduce $\frac{165}{440}$ to its lowest terms.

$$\begin{array}{l} 165)440(2 \\ \quad\;\; 330 \\ \quad\;\; \overline{110)165(1} \\ \qquad\quad\; 110 \\ \qquad\quad\; \overline{\;55)110(2} \\ \qquad\qquad\;\; 110 \end{array}$$

The G. C. F. is 55 ;

then 55)$\frac{165}{440} = \frac{3}{8}$. Result.

Or, we might, without using the Rule for finding the G. C. F., divide both terms successively by 5 and 11, thus : $\frac{165}{440} = \frac{33}{88} = \frac{3}{8}$.

EXERCISE XXXV.

Reduce to lowest terms :

1. $\frac{6}{8}$. 2. $\frac{12}{20}$. 3. $\frac{15}{25}$. 4. $\frac{18}{27}$.
5. $\frac{33}{77}$. 6. $\frac{39}{65}$. 7. $\frac{135}{225}$. 8. $\frac{175}{500}$.
9. $\frac{117}{221}$. 10. $\frac{95}{152}$. 11. $\frac{875}{1000}$. 12. $\frac{111}{185}$.
13. $\frac{292}{365}$. 14. $\frac{1236}{1632}$. 15. $\frac{837}{2268}$. 16. $\frac{10395}{16819}$.

17. If a dollar be denoted by 1, express in lowest terms the values of 20 cents, 35 cents, 75 cents and 80 cents.

18. If a battalion of 750 men be denoted by 1, denote in lowest terms a company of 75 men.

19. If a quire of paper (24 sheets) be denoted by 1, denote in lowest terms 15 sheets.

20. If 1 denotes a year (12 months), what fraction in lowest terms will denote 8 months?

21. If 1 denotes a day (24 hours), what fraction in lowest terms will denote school time, from 9 a.m. to 3 p.m.?

22. If 1 represents an hour (60 minutes), what fraction will most simply denote 45 minutes?

23. Express in lowest terms as fractions of a year (365 days), 73 days, 146 days and 292 days, respectively.

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108. Although a fraction is simplest when it is in its lowest terms, yet it is often necessary to raise the terms by multiplying both by the same number.

Example 1. To compare $\frac{3}{5}$ and $\frac{4}{7}$.

$$\frac{3}{5} = \frac{21}{35}, \text{ and } \frac{4}{7} = \frac{20}{35}. \quad (79)$$

We now see that $\frac{3}{5}$ is greater than $\frac{4}{7}$ by $\frac{1}{35}$.

Example 2. To find the sum of $\frac{1}{2}$ and $\frac{1}{3}$.

$$\frac{1}{2} = \frac{3}{6}, \frac{1}{3} = \frac{2}{6}, \text{ and } \frac{3}{6} + \frac{2}{6} = \frac{5}{6}.$$

In these examples we change the given fractions into equivalent fractions with a common denominator.

Since the common denominator must contain all the other denominators, it will be the least possible when it is their L. C. M.

109. To convert two or more fractions to equivalent fractions with the least common denominator.

Rule. Find the L. C. M. of the denominators for the common denominator.

Then the new numerators will be found by dividing the common denominator by the denominator of each fraction, and multiplying the quotient by the numerator.

EXAMPLE. To convert $\frac{5}{8}$, $\frac{7}{12}$, $\frac{9}{16}$ to equivalent fractions with the least common denominator.

$$\begin{array}{r|l} 4 & \cancel{8},\ 12,\ 16 \\ \hline & \quad\ \ 3,\ \ \ 4 \end{array} \qquad \text{L. C. M.} = 4 \times 3 \times 4 = 48.$$

Since $48 = 8 \times 6 = 12 \times 4 = 16 \times 3$, the new numerators will be 5×6, 7×4, 9×3.

Thus we get the equivalent fractions, $\frac{30}{48}$, $\frac{28}{48}$, and $\frac{27}{48}$.

110. To compare two or more fractions.

Rule. Convert them to equivalent fractions, having the least common denominator, and compare the new numerators.

EXAMPLE. To compare $\frac{3}{8}$, $\frac{5}{12}$, $\frac{5}{16}$.

As before, the L. C. M. of 8, 12, 16 = 48; then $\frac{3}{8} = \frac{18}{48}$, $\frac{5}{12} = \frac{20}{48}$, $\frac{5}{16} = \frac{15}{48}$. Arranging the given fractions in descending order, we have $\frac{5}{12}$, $\frac{3}{8}$, $\frac{5}{16}$.

Note. The given fractions must be in lowest terms, for if they are not, the L. C. M. of the denominators will not always be the *least* common denominator.

Thus, if the fractions be $\frac{6}{9}$, $\frac{4}{8}$, $\frac{9}{12}$, the L. C. M. of 9, 8, 12 is 72; but the given fractions when reduced to lowest terms are $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, which can be converted to $\frac{8}{12}$, $\frac{6}{12}$, $\frac{9}{12}$.

EXERCISE XXXVI.

Reduce to equivalent fractions with least common denominator:

1. $\frac{2}{3}$ and $\frac{3}{4}$.
2. $\frac{5}{6}$ and $\frac{3}{8}$.
3. $\frac{1}{3}$ and $\frac{1}{5}$.
4. $\frac{5}{12}$ and $\frac{4}{9}$.
5. $\frac{3}{5}$ and $\frac{5}{4}$.
6. $\frac{1}{2}$ and $\frac{5}{11}$.

7. $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$.

8. $\frac{2}{5}$, $\frac{3}{10}$ and $\frac{7}{15}$.

9. $\frac{1}{3}$, $\frac{1}{5}$ and $\frac{1}{7}$.

10. $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{6}{7}$.

11. $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{6}$ and $\frac{7}{12}$.

12. $\frac{2}{15}$, $\frac{7}{10}$, $\frac{5}{6}$ and $\frac{1}{3}$.

13. $\frac{13}{18}$, $\frac{17}{24}$, $\frac{11}{30}$ and $\frac{21}{32}$.

14. $\frac{5}{28}$, $\frac{3}{14}$, $\frac{8}{21}$ and $\frac{5}{12}$.

15. $\frac{2}{3}$, $\frac{4}{9}$, $\frac{1}{12}$ and $\frac{7}{18}$.

16. $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{5}$ and $\frac{4}{7}$.

17. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$ and $\frac{9}{16}$.

18. $\frac{3}{5}$, $\frac{4}{25}$, $\frac{7}{125}$ and $\frac{11}{625}$.

19. $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$ and $\frac{1}{81}$.

20. $\frac{9}{10}$, $\frac{7}{100}$, $\frac{3}{1000}$ and $\frac{1}{10000}$.

Compare and arrange in descending order :

1. $\frac{3}{8}$, $\frac{5}{12}$ and $\frac{2}{3}$.

2. $\frac{5}{9}$, $\frac{7}{12}$ and $\frac{11}{18}$.

3. $\frac{3}{10}$, $\frac{1}{3}$, $\frac{2}{5}$ and $\frac{1}{2}$.

4. $\frac{5}{11}$, $\frac{3}{7}$ and $\frac{9}{14}$.

5. Two-thirds, three-fourths and five-sixths.

6. One-third, two-fifths and three-sevenths.

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ADDITION OF FRACTIONS.

111. To find the sum of $\frac{1}{12}$, $\frac{5}{12}$ and $\frac{7}{12}$.

Since in all the fractions the unit is divided into 12 equal parts, a part has the same value in all, and ∴ we can add the numbers of parts and say, $1 + 5 + 7 = 13$.

Or, $\frac{1}{12} + \frac{5}{12} + \frac{7}{12} = \dfrac{1 + 5 + 7}{12} = \frac{13}{12} = 1\frac{1}{12}$.

If we have to find the sum of $\frac{3}{4}$ and $\frac{5}{6}$, we must not add together 3 and 5, ∵ a part in one is not the same as a part in the other. But we can change $\frac{3}{4}$ into $\frac{9}{12}$, and $\frac{5}{6}$ into $\frac{10}{12}$, and now we can take the sum of 9 parts and 10 parts. Or, $\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \dfrac{9 + 10}{12} = \frac{19}{12} = 1\frac{7}{12}$.

Hence,

112. To add fractions:

Rule.—If the given fractions have not already a common denominator, convert them to equivalent fractions with the least common denominator.

Take the sum of the new numerators for the numerator of the result, and the common denominator for the denominator.

Note 1. Before applying the rule, improper fractions should be reduced to mixed numbers, and all fractions should be in lowest terms.

Note 2. If there be any mixed numbers, the whole numbers should be added separately.

Note 3. If the result be an improper fraction, it should be reduced to a mixed number, and the fractional part should be in lowest terms.

Example 1. Find the sum of $\frac{5}{8}$, $\frac{1}{6}$ and $\frac{7}{12}$.

The L. C. M. of 8, 6 and 12 = 24.

$$\text{Then } \tfrac{5}{8} + \tfrac{1}{6} + \tfrac{7}{12} = \tfrac{15}{24} + \tfrac{4}{24} + \tfrac{14}{24}$$

$$= \frac{15 + 4 + 14}{24} = \tfrac{33}{24} = \tfrac{11}{8} = 1\tfrac{3}{8}.$$

Example 2. Add together $7\frac{2}{5}$, $9\frac{3}{10}$ and $\frac{8}{15}$.

$$7\tfrac{2}{5} + 9\tfrac{8}{10} + \tfrac{8}{15} = 7 + 9 + \tfrac{2}{5} + \tfrac{3}{10} + \tfrac{8}{15}$$

$$= 16 + \tfrac{12}{30} + \tfrac{9}{30} + \tfrac{16}{30}$$

$$= 16 + \tfrac{37}{30} = 16 + 1\tfrac{7}{30} = 17\tfrac{7}{30}.$$

EXERCISE XXXVII.

Find the result of

1. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$.
2. $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$.
3. $\frac{1}{3} + \frac{1}{6} + \frac{1}{9}$.
4. $\frac{3}{8} + \frac{7}{10} + \frac{1}{5}$.
5. $\frac{4}{15} + \frac{5}{12} + \frac{2}{3}$.
6. $\frac{3}{5} + \frac{4}{7}$.

7. $\frac{4}{9} + \frac{7}{12} + \frac{5}{8}$.

8. $\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$.

9. $7\frac{1}{8} + 5\frac{3}{8} + 3\frac{1}{2}$.

10. $156\frac{7}{12} + 89\frac{3}{8}$.

Find the sum of

11. $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$ and $4\frac{1}{5}$.

12. $87\frac{3}{4}$, $19\frac{1}{6}$, and $235\frac{8}{10}$.

13. $48\frac{5}{14}$, $\frac{4}{7}$, $26\frac{8}{21}$, and $\frac{11}{28}$.

14. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, and $\frac{1}{64}$.

15. $\frac{2}{3}$, $\frac{8}{9}$, $\frac{26}{27}$, and $\frac{80}{81}$.

16. $\frac{26}{35}$, $\frac{13}{42}$, $\frac{8}{63}$, and $\frac{5}{18}$.

17. Which is the greater, the sum of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$, or the sum of $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{6}$?

18. Shew that a person who has only one section of land can not give his four sons a half, a third, a quarter, and a sixth of a section respectively. Could he, if he had two sections?

19. Compare the result of $\frac{3}{4} + \frac{5}{6} + \frac{7}{8}$ with that of $\frac{2}{3} + \frac{5}{6} + \frac{8}{9}$.

20. Add together $8\frac{11}{56}$, $17\frac{5}{24}$, $28\frac{7}{30}$, and $469\frac{18}{42}$.

21. Shew that the sum of $\frac{2}{3}$ and $\frac{3}{2}$ is greater than 2.

22. Shew that $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$ falls short of 1 by $\frac{1}{32}$.

SUBTRACTION OF FRACTIONS.

113. To subtract $\frac{3}{16}$ from $\frac{11}{16}$ we can at once take 3 parts from 11 parts, and find the remainder to be 8 parts.

$$\text{Or, } \frac{11}{16} - \frac{3}{16} = \frac{8}{16} = \frac{1}{2}.$$

But if we wish to subtract $\frac{3}{8}$ from $\frac{5}{6}$, we cannot subtract 3 parts from 5 parts; ∵ in one fraction the unit is divided into 8 equal parts, and in the other into 6 equal parts, so that a part in one is not equal to a part in the other. We know (79), however, that $\frac{3}{8} = \frac{9}{24}$ and $\frac{5}{8} = \frac{20}{24}$, and now we can subtract 9 parts from 20 parts, and find the remainder to be 11 parts.

$$\text{Or, } \frac{5}{6} - \frac{3}{8} = \frac{20}{24} - \frac{9}{24} = \frac{20-9}{24} = \frac{11}{24}.$$

Hence we get the following Rule :

114. To subtract one fraction from another:

Rule. If the fractions have not already a common denominator, reduce them to equivalent fractions with least common denominator, and then take the difference of the numerators for the numerator of the result and the common denominator for the denominator.

Example 1. From $\frac{7}{12}$ take $\frac{3}{8}$.

$$\frac{7}{12} - \frac{3}{8} = \frac{14}{24} - \frac{9}{24} = \frac{14-9}{24} = \frac{5}{24}.$$

Note 1. In the case of mixed numbers, the difference of the whole numbers should be taken first. Also, it is generally better to reduce improper fractions to mixed numbers.

Note 2. If the result be an improper fraction, it should be reduced to a mixed number, and the fraction of the result should be in lowest terms.

Example 2. From $12\frac{17}{21}$ subtract $5\frac{11}{28}$.

$$\begin{aligned} 12\tfrac{17}{21} - 5\tfrac{11}{28} &= 7 + \tfrac{68}{84} - \tfrac{33}{84} \\ &= 7\tfrac{35}{84} = 7\tfrac{5}{12}. \end{aligned}$$

Example 3. Find the difference between $26\frac{5}{12}$ and $8\frac{7}{9}$.

$$\begin{aligned} 26\tfrac{5}{12} - 8\tfrac{7}{9} &= 18 + \tfrac{15}{36} - \tfrac{28}{36} \\ &= 17 + \tfrac{36}{36} + \tfrac{15}{36} - \tfrac{28}{36} \\ &= 17 + \frac{51-28}{36} = 17\tfrac{23}{36}. \end{aligned}$$

Since 28 parts cannot be taken from 15 parts, we increase the 15 parts by a unit, $\frac{36}{36}$, taken from the 18 units.

Or we may subtract the $\frac{28}{36}$ from a unit, $\frac{36}{36}$, which leaves $\frac{8}{36}$, and then adding in the $\frac{15}{36}$, we get $\frac{23}{36}$, or, altogether, $17\frac{23}{36}$.

EXERCISE XXXVIII.

Find the result of

1. $\frac{1}{2} - \frac{1}{3}$.
2. $\frac{1}{3} - \frac{1}{4}$.
3. $\frac{1}{5} - \frac{1}{7}$.
4. $\frac{5}{6} - \frac{3}{4}$.
5. $8 - \frac{3}{7}$.
6. $\frac{5}{12} - \frac{3}{10}$.
7. $\frac{13}{14} - \frac{1}{2}$.
8. $5\frac{2}{3} - 2\frac{1}{2}$.
9. $8\frac{4}{9} - 1\frac{5}{6}$.
10. $9\frac{7}{12} - 6\frac{5}{18}$.
11. $7\frac{3}{10} - 4\frac{8}{15}$.
12. $20 - 4\frac{5}{8}$.
13. $16\frac{5}{9} - 12\frac{3}{9}$.
14. $18\frac{11}{25} - 6\frac{7}{15}$.
15. $\frac{118}{180} - \frac{121}{225}$.

16. What is the difference between $\frac{7}{9} + \frac{5}{12}$ and $\frac{7 + 5}{9 + 12}$?

17. Two-thirds of a certain sum of money exceeds three-fifths of it by $1.50. How much is the sum?

18. A does half of a certain work and B a third of it; if C finishes it, how much will he do?

19. How much does the sum of $\frac{5}{8}$ and $\frac{3}{10}$ exceed the difference of these fractions?

20. How much must be added to the sum of $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{8}$ to make a unit?

MULTIPLICATION OF FRACTIONS.

115. Fractions are multiplied, in the proper sense of the word, in the same manner as whole numbers. Thus:

$\frac{3}{5} \times 2 = \frac{6}{5}$ (3 parts taken twice = 6 parts).

$\frac{5}{8} \times 3 = \frac{15}{8}$ (5 parts taken 3 times = 15 parts

But since we denote the value of 5 pounds at 8c. a pound by 8c. × 5, it is convenient also to denote the value of half a pound by 8c. × $\frac{1}{2}$, and to call the operation multiplication, although, instead of taking 8c. a number of times, we have to take $\frac{1}{2}$ of 8c., or 4c.

So 15 × $\frac{2}{3}$ means $\frac{2}{3}$ of 15, and the operation is to divide 15 into 3 equal parts and to take two parts. Thus:

$$\frac{1}{3} \text{ of } 15 = 5,$$
$$\therefore \frac{2}{3} \text{ of } 15 = 10.$$

116. To prove that $\frac{4}{5} \times \frac{2}{3}$ or $\frac{4}{5}$ of $\frac{2}{3} = \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$.

$\frac{1}{3}$ of $\frac{4}{5} = \frac{1}{3}$ of $\frac{12}{15} = \frac{4}{15}$ ($\frac{1}{3}$ of 12 parts = 4 parts),

$\therefore \frac{2}{3}$ of $\frac{4}{5} = \frac{4}{15} \times 2 = \frac{8}{15}$ (4 parts taken twice).

Note. Hence it will be seen that the value of $\frac{4}{5} \times \frac{2}{3}$ is the same whether it be taken as $\frac{4}{5}$ of $\frac{2}{3}$ or $\frac{2}{3}$ of $\frac{4}{5}$.

117. Cancelling. Since the value of a fraction is not altered by dividing both terms by the same number, the work may be shortened by striking out from the numerator and denominator of the result any factor that is common to both.

EXAMPLE. Find the result of $\frac{5}{6}$ of $\frac{9}{10}$.

$$\frac{5}{6} \text{ of } \frac{9}{10} = \frac{5 \times 9}{6 \times 10} = \frac{\cancel{5} \times \cancel{3} \times 3}{2 \times \cancel{3} \times 2 \times \cancel{5}} = \frac{3}{4}.$$

$$\text{Or, } \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{6}}} \text{ of } \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{10}}} = . \quad \text{Hence,}$$

118. To multiply one fraction by another, or to reduce a compound fraction to a simple fraction.

Rule. Take the product of the numerators for the numerator of the result, and the product of the denominators for the denominator.

Note 1. Mixed numbers must be reduced to improper fractions before applying the rule; and if the result is an improper fraction, it should be reduced to a mixed number.

Note 2. The work should be shortened as much as possible by cancelling, and the result should be reduced to lowest terms.

Example 1. Find the result of $2\frac{2}{3}$ of $3\frac{3}{4}$.

$$2\tfrac{2}{3} \text{ of } 3\tfrac{3}{4} = \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{3}}} \text{ of } \frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{4}}} = \frac{2 \times 5}{1 \times 1} = \tfrac{10}{1} = 10.$$

Example 2. Multiply the sum of $\frac{3}{8}$ and $\frac{1}{6}$ by their difference.

$$\tfrac{3}{8} + \tfrac{1}{6} = \frac{9+4}{24} = \tfrac{13}{24}, \text{ and } \tfrac{3}{8} - \tfrac{1}{6} = \frac{9-4}{24} = \tfrac{5}{24},$$

$$\text{then } \tfrac{13}{24} \times \tfrac{5}{24} = \frac{13 \times 5}{24 \times 24} = \tfrac{65}{576}.$$

Example 3. A gives B $\frac{1}{5}$th of his cake, and C $\frac{3}{8}$ths of the remainder. Compare the shares of A, B and C.

B's share = $\frac{1}{5}$ and A has $\frac{4}{5}$ left.
C's " = $\frac{3}{8}$ of $\frac{4}{5} = \frac{3}{10}$.
A's " = $\frac{5}{8}$ of $\frac{4}{5} = \frac{1}{2}$.

Reducing to equivalent fractions with least common denominator.

A's = $\frac{1}{2} = \frac{5}{10}$, B's = $\frac{3}{10}$, C's = $\frac{1}{5} = \frac{2}{10}$.

Example 4. A's age is $\frac{4}{9}$ths of C's, and B's is $\frac{5}{6}$ths of C's. What fraction of B's is A's age?

$\frac{5}{6}$ths of C's = B's.
∴ $\frac{1}{6}$th “ = $\frac{1}{5}$ of B's.
∴ $\frac{6}{6}$ths or C's = $\frac{6}{5}$ of B's.
∴ A's = $\frac{4}{9}$ of $\frac{6}{5}$ of B's = $\frac{8}{15}$ of B's.

EXERCISE XXXIX.

Find the result of

1. $\frac{2}{3}$ of 5. 2. $\frac{1}{4}$ of 3. 3. $\frac{2}{5}$ of 5.
4. $\frac{9}{10}$ of 4. 5. $\frac{5}{12}$ of 6. 6. $\frac{2}{3}$ of 21.
7. $\frac{4}{9}$ of $\frac{3}{8}$. 8. $\frac{8}{9}$ of $\frac{15}{16}$. 9. $\frac{20}{21}$ of $\frac{14}{25}$.
10. $2\frac{1}{4} \times 3\frac{1}{3}$. 11. $7\frac{1}{2} \times 3\frac{1}{5}$. 12. $4\frac{3}{8} \times 5\frac{1}{7}$.
13. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$. 14. $1\frac{1}{2}$ of $1\frac{1}{2}$ of $1\frac{1}{2}$.
15. $\frac{8}{9}$ of $\frac{7}{10}$ of $6\frac{3}{4}$. 16. $5\frac{3}{5} \times 3\frac{4}{7} \times \frac{3}{40}$.

17. If A spends $\frac{4}{9}$ of $\frac{3}{10}$ of his money, what part has he left?

18. If A spends $\frac{1}{6}$th of his money on cakes and $\frac{2}{5}$ths of the remainder on oranges, what part has he left?

19. A does $\frac{1}{4}$th of a certain work, and B does $\frac{2}{3}$rds of the remainder. How much is left to be done?

20. A eats $\frac{1}{10}$th of his cake and gives away $\frac{3}{8}$ths of it, while B eats $\frac{1}{10}$th of his cake and gives away $\frac{3}{8}$ths of the remainder. Compare the portions remaining to A and B.

21. A, who owned $\frac{5}{9}$ths of a farm, sold $\frac{6}{11}$ths of his share to B, who sold $\frac{22}{45}$ths of what he bought to C, what part of the farm did C buy?

22. The population of a town increased by $\frac{1}{4}$th of itself each year. How much less than double did it become in 3 years?

23. A and B have equal sums at first. A increases his money each year by $\frac{1}{5}$th of the original amount; B augments his each year by $\frac{1}{5}$th of what it was at the beginning of the year. How much more will A's money be at the end of 4 years than B's at the end of 3 years?

24. A lost one-fifth of his money and spent one-fourth of the remainder, and then had 45 cents left. How much had he at first?

25. Divide $1.75 between A and B, so that A's share shall be 10 cents more than $\frac{3}{8}$ths of B's share.

DIVISION OF FRACTIONS.

119. To divide $\frac{8}{11}$ by 2:

Here we can divide just as in whole numbers.

$$\tfrac{8}{11} \div 2 = \tfrac{4}{11} \quad (8 \text{ parts} \div 2 = 4 \text{ parts}).$$

To divide $\frac{8}{11}$ by 3:

If we proceed as before,

$$\tfrac{8}{11} \div 3 = \frac{2\frac{2}{3}}{11} \quad (8 \text{ parts} \div 3 = 2\tfrac{2}{3} \text{ parts}).$$

But we can get a simple result by dividing the unit into 33 equal parts.

$$\text{then } \tfrac{8}{11} \div 3 = \tfrac{24}{33} \div 3 = \tfrac{8}{33} = \frac{8}{11 \times 3}. \quad (79)$$

Or we might say,

$$\tfrac{8}{11} \div 3 = \tfrac{1}{3} \text{ of } \tfrac{8}{11} = \tfrac{8}{33}.$$

Hence,

120. To divide a fraction by a whole number:

Rule. Divide the numerator, or multiply the denominator by the number.

121. To divide $\frac{7}{8}$ by $\frac{3}{5}$:

Since $\frac{3}{5}$ is $\frac{1}{5}$ of 3, the quotient $\frac{7}{8} \div \frac{3}{5}$ should be 5 times as much as $\frac{7}{8} \div 3$.

$$\text{Now } \frac{7}{8} \div 3 = \frac{1}{3} \text{ of } \frac{7}{8} = \frac{7}{24}$$

$$\therefore \frac{7}{8} \div \frac{3}{5} = \frac{7}{24} \times 5 = \frac{35}{24} = \frac{7}{8} \times \frac{5}{3}.$$

$$\text{So } \frac{6}{13} \div \frac{3}{13} = \frac{6}{\cancel{13}} \times \frac{\cancel{13}}{\cancel{3}} = 2$$

a result that might be obtained as in whole numbers. (In 6 parts 3 parts are contained twice).

To divide 5 by $\frac{2}{3}$:

$$5 \div 2 = \frac{1}{2} \text{ of } 5 = \frac{5}{2}$$

$$\therefore 5 \div \frac{2}{3} = \frac{5}{2} \times 3 = \frac{15}{2} = \frac{5}{1} \times \frac{3}{2}.$$

Hence,

122. To divide a fraction by a fraction:

Rule. Invert the divisor and proceed as in multiplication.

Note 1. This Rule includes division of a fraction by a whole number, and division of a whole number by a fraction.

EXAMPLE. $\frac{5}{8} \div 3 = \frac{5}{8} \div \frac{3}{1} = \frac{5}{8} \times \frac{1}{3} = \frac{5}{24}$.

Note 2. Before applying the Rule, the dividend and divisor must both be reduced to simple fractions.

Note 3. The result should be reduced to lowest terms, and, when an improper fraction, to a mixed number.

Note 4. As in multiplication the work should be shortened as much as possible by cancelling.

EXAMPLE 1. To divide $\frac{8}{15}$ by $\frac{4}{9}$.

$$\frac{8}{15} \div \frac{4}{9} = \frac{\overset{2}{\cancel{8}}}{\underset{5}{\cancel{15}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{4}}} = \frac{6}{5} = 1\frac{1}{5}.$$

Example 2. Find the result of $3\frac{1}{3} \div 1\frac{7}{8}$.

$$3\tfrac{1}{3} \div 1\tfrac{7}{8} = \tfrac{10}{3} \div \tfrac{15}{8} = \frac{\overset{2}{\cancel{10}}}{3} \times \frac{8}{\underset{3}{\cancel{15}}} = \tfrac{16}{9} = 1\tfrac{7}{9}.$$

Example 3. Simplify $\frac{4}{15}$ of $3\frac{2}{11} \div 1\frac{5}{9}$.

$$\tfrac{4}{15} \text{ of } 3\tfrac{2}{11} \div \tfrac{15}{9} = \frac{4}{\underset{3}{\cancel{15}}} \text{ of } \frac{\overset{7}{\cancel{35}}}{11} \div \tfrac{14}{9} = \frac{4 \times 7}{3 \times 11} \times \tfrac{9}{14} =$$

$$\frac{\overset{2}{\cancel{4}} \times \cancel{7} \times \overset{3}{\cancel{9}}}{\cancel{3} \times 11 \times \underset{2}{\cancel{14}}} = \tfrac{6}{11}.$$

Example 4. If $\frac{2}{3}$rds of a pound cost $\frac{5}{7}$ths of a dollar, how much will $\frac{3}{4}$ths of a pound cost?

The value of $\frac{2}{3}$ of a pound = $\frac{5}{7}$ of a dollar.
∴ " $\frac{1}{3}$ " = $\frac{1}{2}$ of $\frac{5}{7}$ of a dollar.
∴ " $\frac{3}{3}$, or 1 pound = $\frac{3}{2}$ of $\frac{5}{7}$ "
∴ " $\frac{3}{4}$ of a pound = $\frac{3}{4}$ of $\frac{3}{2}$ of $\frac{5}{7}$ of a dollar.
= $\frac{45}{56}$ of a dollar.

Example 5. If 4 men do $\frac{3}{5}$ths of a certain work in 1 day, in what time will 6 men do $\frac{7}{8}$ths of the same work?

4 men do $\frac{3}{5}$ of work in 1 day
∴ 1 man does $\frac{3}{5}$ " " " 4 days
1 " $\frac{1}{5}$ " " " $\frac{4}{3}$ "
1 " $\frac{5}{5}$, or the whole " $\frac{20}{3}$ "
1 " $\frac{7}{8}$ of work " $\frac{7}{8}$ of $\frac{20}{3}$ days
6 men do $\frac{7}{8}$ " " $\frac{1}{6}$ of $\frac{7}{8}$ of $\frac{20}{3}$ days

$$\text{and } \tfrac{1}{6} \text{ of } \frac{7}{\underset{2}{\cancel{8}}} \text{ of } \frac{\overset{5}{\cancel{20}}}{3} = \tfrac{35}{36} \text{ day.}$$

EXERCISE XL.

Find the result of

1. $\frac{9}{14} \div 3$.
2. $\frac{7}{10} \div 2$.
3. $1\frac{7}{8} \div 5$.
4. $2\frac{1}{4} \div 6$.
5. $\frac{3}{5} \div \frac{9}{10}$.
6. $\frac{1}{9} \div \frac{1}{12}$.
7. $\frac{9}{16} \div 3\frac{3}{4}$.
8. $3\frac{1}{8} \div 7\frac{1}{2}$.
9. $\frac{5}{12}$ of $\frac{8}{15} \div \frac{4}{9}$.
10. $\frac{324}{325} \div \frac{108}{175}$.
11. $15\frac{1}{6} \div 12\frac{3}{5}$.
12. $\frac{8}{21}$ of $\frac{5}{6} \div 3\frac{4}{7}$.

13. Divide the sum of $\frac{5}{6}$ and $\frac{1}{4}$ by their difference.

14. If the dividend be $\frac{8}{21}$ and quotient $\frac{6}{7}$, what is the divisor?

15. If the divisor be $\frac{9}{16}$ and the quotient $\frac{20}{21}$, what is the dividend?

16. A sold a watch to B, who sold it to C for $\frac{1}{5}$th more than he gave for it. C paid for the watch $\frac{1}{3}$ more than A did. How much did A gain?

17. If $\frac{3}{8}$ths of a fish weigh $\frac{9}{10}$ths of a pound, what will the whole weigh?

18. What fraction multiplied by $\frac{8}{15}$ will produce $\frac{12}{25}$?

19. If $\frac{3}{4}$ of A's money is $\frac{5}{6}$ of B's, what fraction of A's money is B's?

20. If 5 men can do a certain work in $\frac{2}{3}$ of a day, in what time could 6 men do the same work?

21. If a man earn \$$1\frac{3}{4}$ a day, in what time will he earn \$$10\frac{1}{2}$?

22. If a man earn \$1 in $\frac{5}{8}$ths of a day, how much will he earn in $7\frac{1}{2}$ days?

23. A boy in one game lost $\frac{1}{5}$th of his marbles, and in another game $\frac{1}{6}$th of the remainder, and then had 48 left. How many had he at first?

24. What fraction of $\frac{8}{15}$ is $\frac{4}{9}$?

SIMPLIFICATION OF FRACTIONAL EXPRESSIONS.

123. Fractional expressions are often met with which require several or all of the preceding operations to reduce them to simple fractions.

The operations to be performed are denoted by the signs +, −, × or "of", and ÷; and the pupil must be careful to note how far the force of each of these signs extends.

I. The parts of an expression connected by + or − are called its terms, and the force of either of these signs applies only to the term immediately following the sign, unless its application is extended by the use of brackets.

Example 1. Simplify $\frac{3}{4} - \frac{1}{6} + \frac{1}{8}$.

Here $\frac{1}{6}$ is to be subtracted from $\frac{3}{4}$ and $\frac{1}{8}$ added to the result. The operations may be carried on simultaneously: thus,

$$\tfrac{3}{4} - \tfrac{1}{6} + \tfrac{1}{8} = \frac{18 - 4 + 3}{24} = \tfrac{17}{24}. \quad \text{Result.}$$

Example 2. Simplify $\frac{3}{4} - (\frac{1}{6} + \frac{1}{8})$.

Here the force of − is extended to the result of the expression enclosed in brackets. This result should be obtained first:

$$\text{thus, } \tfrac{1}{6} + \tfrac{1}{8} = \frac{4 + 3}{24} = \tfrac{7}{24}$$

$$\text{then, } \tfrac{3}{4} - \tfrac{7}{24} = \frac{18 - 7}{24} = \tfrac{11}{24}. \quad \text{Result.}$$

II. A term may be composed of several factors connected by the signs × or "of" and ÷, and the result of such a term must be found before adding or subtracting.

Example 1. Simplify $\frac{1}{2}$ of $\frac{2}{3} + \frac{2}{3}$ of $\frac{1}{4} - \frac{3}{4}$ of $\frac{2}{5}$.

First, $\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{3}$, $\frac{2}{3}$ of $\frac{1}{4} = \frac{1}{6}$, and $\frac{3}{4}$ of $\frac{2}{5} = \frac{3}{10}$

Then, $\frac{1}{3} + \frac{1}{6} - \frac{3}{10} = \dfrac{10 + 5 - 9}{30} = \frac{6}{30} = \frac{1}{5}$. Result.

III. "Of" and × apply only to the factors between which they stand, and ÷ applies only to the factor immediately following it.

Example 1. Simplify $\frac{7}{12} - \frac{3}{10}$ of $\frac{5}{6}$.

First, $\frac{3}{10}$ of $\frac{5}{6} = \frac{1}{4}$.

Then, $\frac{7}{12} - \frac{1}{4} = \dfrac{7 - 3}{12} = \frac{4}{12} = \frac{1}{3}$. Result.

As before, the application of these signs may be extended by the use of brackets.

Example 2. $(\frac{7}{12} - \frac{3}{10})$ of $\frac{5}{6}$.

Here the result of $\frac{7}{12} - \frac{3}{10}$ is to be taken as a factor.

First, $\frac{7}{12} - \frac{3}{10} = \dfrac{35 - 18}{60} = \frac{17}{60}$.

Then, $\frac{17}{60}$ of $\frac{5}{6} = \frac{17}{72}$. Result.

Example 3. $\frac{1}{2} + \frac{3}{5} \div \frac{9}{10} - \frac{5}{6}$.

First, $\frac{3}{5} \div \frac{9}{10} = \frac{3}{5} \times \frac{10}{9} = \frac{2}{3}$.

Then, $\frac{1}{2} + \frac{2}{3} - \frac{5}{6} = \dfrac{3 + 4 - 5}{6} = \frac{2}{6} = \frac{1}{3}$. Result.

Example 4. $(\frac{1}{2} + \frac{3}{5}) \div (\frac{9}{10} - \frac{5}{6})$.

Here the result of $\frac{1}{2} + \frac{3}{5}$ is to be divided by the result of $\frac{9}{10} - \frac{5}{6}$.

$\frac{1}{2} + \frac{3}{5} = \dfrac{5 + 6}{10} = \frac{11}{10}$; $\frac{9}{10} - \frac{5}{6} = \dfrac{27 - 25}{30} = \frac{2}{30} = \frac{1}{15}$.

Then, $\frac{11}{10} \div \frac{1}{15} = \frac{11}{10} \times \frac{15}{1} = \frac{33}{2} = 16\frac{1}{2}$. Result.

Example 5. Simplify $\frac{5}{6} \div 1\frac{1}{9} \div 7\frac{1}{2}$.

Here $\frac{5}{6}$ is to be divided by $1\frac{1}{9}$ and the result by $7\frac{1}{2}$. The operations may be carried on simultaneously, thus:

$$\frac{5}{6} \div 1\frac{1}{9} \div 7\frac{1}{2} = \frac{5}{6} \div \frac{10}{9} \div \frac{15}{2} = \frac{5}{6} \times \frac{9}{10} \times \frac{2}{15} = \frac{1}{10}.$$ Result.

Example 6. Simplify $\frac{5}{6} \div (1\frac{1}{9} \div 7\frac{1}{2})$.

Here $\frac{5}{6}$ is to be divided by the result of $1\frac{1}{9} \div 7\frac{1}{2}$.

First, $1\frac{1}{9} \div 7\frac{1}{2} = \frac{10}{9} \div \frac{15}{2} = \frac{10}{9} \times \frac{2}{15} = \frac{4}{27}$.

Then, $\frac{5}{6} \div \frac{4}{27} = \frac{5}{6} \times \frac{27}{4} = \frac{45}{8} = 5\frac{5}{8}$. Result.

Example 7. Simplify $\frac{1}{5}$ of $\left\{3 - \frac{2}{3} \text{ of } (2 - \frac{1}{6} \text{ of } 7\frac{1}{2})\right\}$

$\frac{1}{6}$ of $7\frac{1}{2} = \frac{1}{6}$ of $\frac{15}{2} = \frac{5}{4} = 1\frac{1}{4}$; $2 - 1\frac{1}{4} = \frac{3}{4}$; $\frac{2}{3}$ of $\frac{3}{4} = \frac{1}{2}$; $3 - \frac{1}{2} = 2\frac{1}{2}$; finally, $\frac{1}{5}$ of $2\frac{1}{2} = \frac{1}{5}$ of $\frac{5}{2} = \frac{1}{2}$. Result.

EXERCISE XLI.

Reduce to simple fractions or mixed numbers:

1. $\frac{3}{4} - \frac{1}{6} - \frac{1}{8}$.
2. $\frac{3}{4} - (\frac{1}{6} - \frac{1}{8})$.
3. $\frac{5}{6}$ of $\frac{9}{10} - \frac{4}{15}$.
4. $\frac{5}{6}$ of $(\frac{9}{10} - \frac{4}{15})$.
5. $\frac{2}{3} + \frac{1}{12} \div \frac{4}{9}$.
6. $(\frac{2}{3} + \frac{1}{12}) \div \frac{4}{9}$.
7. $\frac{1}{2} + \frac{2}{3}$ of $\frac{9}{10} - \frac{5}{6}$.
8. $(\frac{1}{2} + \frac{2}{3})$ of $(\frac{9}{10} - \frac{5}{6})$.
9. $\frac{1}{2} + \frac{2}{3}$ of $(\frac{9}{10} - \frac{5}{6})$.
10. $(\frac{1}{2} + \frac{2}{3})$ of $\frac{9}{10} - \frac{5}{6}$.
11. $\frac{4}{5}$ of $\frac{7}{8} + \frac{5}{6}$ of $\frac{8}{25}$.
12. $(\frac{1}{2} + \frac{1}{3}) \div (\frac{1}{2} - \frac{1}{3})$.
13. $(1\frac{1}{2} + 2\frac{2}{3} + 3\frac{3}{4}) \div 4\frac{2}{9}$.
14. $15\frac{3}{8} \div (\frac{5}{12} + \frac{2}{9}$ of $1\frac{1}{5})$.
15. $\frac{4}{21}$ of $(3\frac{5}{8} - 2\frac{1}{6}) \div \frac{25}{36}$.
16. $\left\{\frac{11}{12} - (\frac{3}{8} + \frac{2}{9})\right\} \div \left\{\frac{11}{12} - \frac{3}{8} + \frac{2}{9}\right\}$.
17. $\left\{3 - \frac{5}{6} \text{ of } (2 - \frac{4}{5} \text{ of } 1\frac{9}{16})\right\} \div \frac{15}{16}$.
18. $\frac{1}{2}$ of $\left\{1 - \frac{2}{3} \text{ of } (1 - \frac{3}{4} \text{ of } \frac{5}{6})\right\}$.

124. Complex Fractions. We have seen before (Art. 73) that a simple fraction denotes a quotient : hence such forms as $\frac{3\frac{1}{3}}{7\frac{1}{2}}$, $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{3}{4}\text{ of }\frac{8}{9}}$, etc., are used to denote that the result of the upper expression is to be divided by that of the lower.

Since the value of a quotient is not altered by multiplying or by dividing the dividend and divisor by the same number, we may use this principle in simplifying complex fractions :

EXAMPLE 1. $\frac{3\frac{1}{3}}{7\frac{1}{2}} = \frac{\frac{10}{3}}{\frac{15}{2}} = \frac{\frac{10}{3}\times 6}{\frac{15}{2}\times 6} = \frac{20}{45} = \frac{4}{9}$. Result.

EXAMPLE 2. $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{3}{4}\text{ of }\frac{8}{9}} = \frac{\frac{5}{6}}{\frac{2}{3}} = \frac{\frac{5}{6}\times 6}{\frac{2}{3}\times 6} = \frac{5}{4} = 1\frac{1}{4}$. Result.

EXAMPLE 3. $\dfrac{1}{2+\dfrac{3}{4+\frac{5}{6}}} = \dfrac{1}{2+\frac{18}{29}} = \frac{29}{76}$. Result.

Here we multiply both terms of $\dfrac{3}{4+\frac{5}{6}}$ by 6, and both terms of $\dfrac{1}{2+\frac{18}{29}}$ by 29.

But in general it is better to treat complex fractions as denoting quotients.

1. $\frac{3\frac{1}{3}}{7\frac{1}{2}} = 3\frac{1}{3} \div 7\frac{1}{2} = \frac{10}{3} \div \frac{15}{2} = \frac{10}{3} \times \frac{2}{15} = \frac{4}{9}$. Result.

2. $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{3}{4}\text{ of }\frac{8}{9}} = (\frac{1}{2}+\frac{1}{3}) \div \frac{3}{4}\text{ of }\frac{8}{9}$

$= \frac{3+2}{6} \div \frac{2}{3} = \frac{5}{6} \times \frac{3}{2} = \frac{5}{4} = 1\frac{1}{4}$. Result.

3. $\dfrac{1}{2+\dfrac{3}{4+\frac{5}{6}}} = 1 \div \left\{2 + 3 \div (4+\frac{5}{6})\right\}$

$3 \div 4\frac{5}{6} = 3 \div \frac{29}{6} = 3 \times \frac{6}{29} = \frac{18}{29}$

$1 \div 2\frac{18}{29} = 1 \div \frac{76}{29} = 1 \times \frac{29}{76} = \frac{29}{76}$.

EXERCISE XLIII.

Find the results of

1. $\dfrac{\frac{24}{25}}{\frac{18}{35}}$ 2. $\dfrac{8\frac{3}{4}}{28}$ 3. $\dfrac{24}{7\frac{1}{5}}$

4. $\dfrac{\frac{5}{8}-\frac{1}{6}}{\frac{5}{8}+\frac{1}{6}}$ 5. $\dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\text{ of }\frac{1}{3}}$ 6. $\dfrac{7\frac{1}{11}\text{ of }4\frac{1}{8}}{\frac{5}{6}-\frac{1}{4}}$

7. $\dfrac{1-(\frac{4}{15}+\frac{3}{10})}{1-\frac{4}{15}+\frac{3}{10}}$ 8. $\dfrac{\frac{3}{5}\text{ of }\frac{12}{13}+\frac{4}{5}\text{ of }\frac{5}{15}}{\frac{4}{5}\text{ of }\frac{12}{13}-\frac{3}{5}\text{ of }\frac{5}{13}}$

9. $\dfrac{\frac{9}{14}\text{ of }(1-\frac{2}{3}\text{ of }\frac{4}{5})}{\frac{9}{14}+\frac{2}{3}\text{ of }\frac{4}{5}}$ 10. $\dfrac{1}{1+\dfrac{1}{1+\frac{1}{2}}}$

11. $\dfrac{5}{4+\dfrac{3}{2+\frac{1}{4}}}$ 12. $\dfrac{\frac{7}{12}-\frac{3}{8}+\frac{5}{6}}{\frac{7}{12}+\frac{3}{8}-\frac{5}{6}}$

125. To express a fraction of a concrete quantity as a compound quantity.

EXAMPLE 1 Find the value of $\frac{7}{12}$ of £13 8*s.* 10½*d.*

$$\begin{array}{r|rrr} & \text{£} & s. & d. \\ 12) & 13 & 8 & 10\frac{1}{2} \\ \hline & 1 & 2 & 4\frac{7}{8} \\ & & & 7 \\ \hline & 7 & 16 & 10\frac{1}{8} \end{array}$$

$\frac{1}{12}$ is found by dividing by 12, and $\frac{7}{12}$ths by taking the quotient 7 times.

EXAMPLE 2. Find the value of $\frac{8}{15}$ of $\frac{25}{44}$ of $7\frac{1}{2}$ acres.

$\frac{8}{15}$ of $\frac{25}{44}$ of $\frac{15}{2}$ acres $= \frac{25}{11}$ acres $= 2\frac{3}{11}$ acres = 2 acres 1r. 3p. 19yds. 2ft. 6in., or

$\frac{3}{11}$ of 1 acre $= \frac{1}{11}$ of 3 acres, the value of which is found by division.

Hence:

Rule. Divide the given quantity by the denominator of the fraction and multiply the quotient by the enumerator.

The fraction or fractional expression must first be reduted to a simple fraction if it be not such already.

Note. We may multiply first and then divide the product.

EXAMPLE 3. After paying $\frac{2}{5}$ths of a debt a person still owes £17 8*s.* 10*d.*, what was the amount of debt at first?

$\frac{2}{5}$ths of debt = £17 8*s.* 10*d.*
∴ $\frac{1}{5}$th " " = £ 8 14*s.* 5*d.*
∴ $\frac{5}{5}$ths " " = £43 12*s.* 1*d.*

EXERCISE XLIII.

Find the value of:

1. $\frac{2}{3}$ of £11 14*s.* 9*d.*
2. $\frac{5}{6}$ of £23 7*s.* 3*d.*
3. $\frac{9}{10}$ of £8 17*s.* 8$\frac{1}{2}$*d.*
4. $\frac{7}{12}$ of £35 10*s.*
5. $\frac{11}{14}$ of a guinea.
6. $\frac{3}{8}$ of 13ac. 1r. 24p.
7. $\frac{4}{11}$ of 7f. 15p.
8. $\frac{17}{36}$ of £21 15*s.*
9. $\frac{13}{16}$ of 1lb. 7oz. 10dwt.
10. $\frac{15}{22}$ of 1 acre.
11. $\frac{7}{18}$ of 6 days 21 hours.
12. $\frac{8}{15}$ of $\frac{19}{14}$ of $22.75.

13. After spending $\frac{3}{8}$ths of my money, I have left $7.50: how much did I spend?

14. A man spends $\frac{1}{4}$th of his money on provisions and $\frac{1}{6}$ on clothes and has £3 5*s.* 4*d.* left: how much had he at first?

15. A butcher bought 5 out of a lot of a dozen sheep and afterwards bought the rest at a price per sheep $\frac{1}{4}$th greater than before. He paid for the whole $68.20. How much did he pay for the first lot?

16. After paying $\frac{1}{6}$ of the rent of a house for taxes, a landlord receives a net rental of $215: how much a month does the tenant pay?

126. To express one quantity as a fraction of another of the same kind.

Example 1. What fraction of \$3.75 is \$1.20 ?

If the unit, or whole, \$3.75, be divided into 375 equal parts, 120 such parts are to be taken ∴ the fraction = $\frac{120}{375} = \frac{8}{25}$.

Example 2. Express 8*s.* 9*d.* as a fraction of £2 5*s.*

£ *s.*	*s.* *d.*
2 5	8 9
20	12
45	105
12	
540	

If the unit, or whole, £2 5*s.*, be divided into 540 equal parts 8*s.* 9*d.* contains 105 such parts.

∴ the required fr. = $\frac{105}{540} = \frac{21}{108} = \frac{7}{36}$.

Hence:

Rule. Reduce both quantities to whole numbers of the same unit.

Take the number in the quantity which is to be a fraction of the other for the numerator and the number in the other for the denominator of the required fraction.

Reduce the fraction to its lowest terms.

EXERCISE XLIV.

1. What fraction of \$5 is \$3.25 ?
2. Express \$6 as a fraction of \$13.50.
3. What fraction is £1 4*s.* of £2 15*s.* ?
4. What fraction of £1 is 18*s.* 10½*d.* ?
5. What fraction of a pound Troy is 5 oz. 7 dwt. 12 grs. ?
6. Express 1 lb. Troy as a fraction of 1 lb. avoirdupois ?
7. Express 1 oz. avoirdupois as a fraction of 1 oz. Troy ?

8. What fraction of a yard is 1 ft. 9 in. ?

9. What fraction of a square foot is the area of a slate 14 inches long, 9 inches wide ?

10. What fraction of a cubic yard is the size of a block of ice 4 ft long, 2 ft. 3 in. wide, 18 in. thick ?

11. A person pays £6 16*s.* 6*d.* out of a debt of £12 17*s.* 10*d.* : what fraction of the debt does he still owe ?

12. What part of a day is 4 h. 45 m. 45 sec. ?

13. A dealer takes off $1.25 for cash from the price of an article marked at $8.75 on credit : what fraction of the price was added for giving credit ?

14. A boy spends 5 hours in school and $2\frac{1}{2}$ hours in preparing his lessons at home : what fraction of the day does he devote to learning ?

15. An oyster can is 6 inches long, $4\frac{1}{2}$ inches broad and $1\frac{7}{8}$ inches deep : what fraction of a quart will it hold, a gallon being $277\frac{1}{4}$ cubic inches ?

16. What is the measure of 15*s.* 9*d.* when the unit is 1 guinea ?

17. Taking the metre as $39\frac{1}{3}$ inches, what will be its measure, if the unit be a yard ?

18. What will be the measure of a yard, if a metre be the unit ?

19. Taking the franc at $9\frac{3}{4}$*d.*, express £1 in francs ; and also 1 franc as a fraction of a pound.

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Miscellaneous Examples.

PAPER I.

1. Give two definitions for a fraction. What is a proper fraction ?

2. Add together $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and from the sum take the difference between $\frac{1}{9}$ and $\frac{1}{11}$.

3. What is the *least* common denominator of the fractions equivalent respectively to

$$\tfrac{2}{3},\ \tfrac{3}{5},\ \tfrac{5}{7},\ \tfrac{7}{9},\ \tfrac{2}{15},\ \tfrac{7}{20}\,?$$

4. Explain how you reduce a fraction to its lowest terms, taking for example $\frac{403}{899}$.

5. What fraction must $\frac{2}{7}$ be multiplied by to give a product $\frac{13}{14}$? Can more than one such fraction be found?

6. From $2\frac{1}{2}$ take $\frac{3}{5}$, and to the result add the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$.

PAPER II.

1. Define the least common multiple of two or more numbers. Show, with reference to the numbers 592, 1369, that the least common multiple is equal to the product of the numbers divided by their greatest common factor.

2. By what fraction does $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$ differ from unity.

3. What is the rule for multiplying one fraction by another. If you multiply two proper fractions together, how will the product compare in value with either of them.

4. Find the value of $\dfrac{3\frac{2}{3}}{12\frac{3}{4}}$ of $\dfrac{1\frac{4}{13} \text{ of } 3\frac{1}{4}}{2\frac{1}{3} \text{ of } 1\frac{2}{9}}$.

5. Simplify $\dfrac{\frac{1}{5} + 2\frac{2}{5} \text{ of } \frac{3}{4}}{5\frac{1}{2} - 1\frac{17}{19}} + \dfrac{11\frac{1}{3} \times 8\frac{1}{6}}{3\frac{2}{5} \times \frac{7}{6}}$.

6. Find value of $2\frac{2}{5} + 3\frac{3}{4}$ of $5\frac{1}{2} - 8\frac{2}{11} \times \frac{11}{30}$.

PAPER III.

1. State how you would divide one vulgar fraction by another. Divide $\frac{3}{8}$ by $\frac{8}{11}$.

2. The length of $\frac{1}{360}$ of the earth's circumference is $69\frac{1}{12}$ English miles: what is the earth's diameter, assuming the diameter of a circle to be $\frac{7}{22}$ of its circumference?

3. Reduce the fractions $\frac{5}{6}$, $\frac{7}{12}$, $\frac{9}{15}$, $\frac{17}{20}$, $\frac{40}{60}$ to equivalent fractions with the *least* common denominator; write the resulting fractions in descending order of value; and take the sum of the greatest and least from the sum of the other three.

4 Simplify $\dfrac{\frac{2}{3} + \frac{4}{5} \text{ of } \frac{5}{9} - \frac{8}{21}}{1 + \frac{2}{3} \times \frac{3}{7} - \frac{5}{9}}$.

5. Simplify $\dfrac{4\frac{1}{2} - 3\frac{1}{3} + 5\frac{1}{12}}{7\frac{1}{2} - 4\frac{1}{3} + 11\frac{1}{12}} - \dfrac{11\frac{3}{4} - 5\frac{7}{15}}{11\frac{3}{4} + 5\frac{7}{15}}$.

6. Of two men, one works regularly 9 hours each day in the week, the other does no work 2 days in the week, but works 5 hours per day for 2 days, and 13 hours per day for the other two. How many days, according to his rate of work, does the former gain in a year?

PAPER IV.

1. Which is the greater of the two fractions $\frac{240}{193}$ and $\frac{1103}{887}$, and by how much?

2 State the rule for adding two fractions. Simplify and add $\dfrac{1}{2 + \dfrac{1}{3 + \frac{1}{4}}}$, $\dfrac{3 + 2}{3\frac{1}{2} - 2\frac{1}{4}}$.

3. Multiply $99\frac{279}{333}$ by $3\frac{2}{3}$, and find the integer nearest to the product.

4. Simplify $\dfrac{3 + \frac{5}{8} \text{ of } \dfrac{21}{7\frac{3}{5}} - \frac{1}{4} - \dfrac{1\frac{2}{3}}{2\frac{1}{2}}}{10 - \frac{151}{228} \text{ of } 5}$.

5. A woman upon returning from the market found that she had in her purse \$1.50, which was a seventh of the money she had spent. What was the money in her purse before going?

6. Two persons buy apples at a cent each: one sells them at 3 for 5 cents, and the other at 5 for 8 cents. Which gains most on \$100, and how much?

PAPER V.

1. State the rule for dividing one fraction by another. What simple fraction is equal to $\frac{2}{3}$ of $3\frac{1}{2}$ of $\dfrac{\frac{\frac{2}{35}}{5}}{7\frac{1}{2}}$?

2. Find the value of $\frac{6}{7}$ of $\frac{4}{9}$ of $2\frac{2}{11}$ of $7\frac{5}{12}$; and reduce $\frac{19395}{22005}$ to its lowest terms.

3. A bankrupt's estate amounts to $915, and his debts to $2745. What can he pay in the dollar? And how much will a creditor lose on a debt of $60?

4. Find the sum of

$$\frac{4\frac{2}{3} - 3\frac{1}{2}}{4\frac{2}{3} + 3\frac{1}{2}} \text{ and } \frac{2}{3} \text{ of } \left(\frac{2}{3} \text{ of } \frac{2\frac{1}{4} + 1\frac{1}{2}}{3\frac{1}{2} - 1} + \frac{1}{2}\right).$$

5. A person, after paying an income tax of 2 cents in the dollar, has a net income of $1176. Find his gross income.

6. When $\frac{1}{4}$th is lost in grinding wheat, a country has to import 80000000 bushels, but can maintain itself on its own produce if only $\frac{1}{20}$th be lost. Find the quantity of wheat grown in the country.

7. Three men were to take to a railway station a pile of cordwood: the first took $\frac{3}{11}$ of it, the second $\frac{3}{7}$, the third took the balance. What fraction of the whole did the last take?

8. Three men were to take a quantity of lumber to a railway station: the first man took $\frac{3}{8}$ of the whole, the second $\frac{9}{11}$ of the remainder, the third $\frac{5}{9}$ of what was then left. Find what fraction of the whole each took away, and how much was left.

9. $\frac{5}{8}$ of a certain estate is good land for raising wheat, $\frac{1}{4}$ for oats and barley, and the rest, 1200 acres, good meadow land. Find the whole number of acres in the estate.

SECTION VII.

DECIMAL FRACTIONS.

127. Arabic Notation extended. If we mark the unit figure by putting after it a full stop (the decimal point), we can then set figures to the right of the unit figure, thus : 73·524. Then on the principle that the value of a figure increases tenfold for every place it is moved to the left, and consequently decreases in a similar manner as it is moved to the right, the 5 will denote $\frac{1}{10}$ of 5 units, or $\frac{5}{10}$, the 2 will denote $\frac{1}{10}$ of $\frac{1}{10}$ of 2 units, or $\frac{2}{100}$, and the 4 will denote $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$ of 4 units, or $\frac{4}{1000}$, so that

$$73{\cdot}524 = 70 + 3 + \tfrac{5}{10} + \tfrac{2}{100} + \tfrac{4}{1000}$$

$$836{\cdot}75 = 800 + 30 + 6 + \tfrac{7}{10} + \tfrac{5}{100}.$$

128. Decimal Fractions. Tenths, hundredths, thousandths, etc., when denoted by an extension of the Arabic Notation, are called Decimal Fractions, or briefly, Decimals.

129. As in whole numbers, ciphers must be used in the absence of significant figures,

$$\text{thus : } 3\tfrac{5}{10} = 3{\cdot}5, \text{ but } 3\tfrac{5}{100} = 3{\cdot}05\text{ ;}$$

$$47 + \tfrac{8}{10} + \tfrac{9}{100} = 47{\cdot}89, \text{ but } 47\tfrac{9}{100} = 47{\cdot}09.$$

$$\text{So } 16 + \tfrac{2}{10} + \tfrac{6}{1000} = 16{\cdot}206, \text{ and } \tfrac{4}{100} = 0{\cdot}04 \text{ or } {\cdot}04.$$

From the above examples it can be seen that counting from the unit figure, the first figure to the left denotes tens, the first to the right, tenths ; the second to the left denotes hundreds, the second to the right, hundredths ; and so on.

130. The expression ·546 can denote either $\frac{5}{10} + \frac{4}{100} + \frac{6}{1000}$ or $\frac{546}{1000}$.

$$\text{For } \frac{5}{50} + \frac{4}{100} + \frac{6}{1000} = \frac{500 + 40 + 6}{1000} = \frac{546}{1000}.$$

Hence ·546 may be called *a* decimal fraction.

The number of figures after the decimal point is the same as the number of 0's in the denominator of the vulgar fraction.

131. Ciphers which do not come between significant figures and the decimal point do not affect the value of the figures and may be omitted. Thus ·600, ·60, ·6, all denote the same value $\frac{6}{10}$.

If we regard ·6 as $\frac{6}{10}$, ·60 as $\frac{60}{100}$, and ·600 as $\frac{600}{1000}$, still these fractions all reduce to $\frac{6}{10}$.

132. Decimals are multiplied by 10 by moving the point one place to the right, and they are divided by 10 by moving the point one place to the left, thus :

$$31{\cdot}56 \times 10 = 315{\cdot}6\;;\quad 27{\cdot}85 \div 10 = 2{\cdot}785$$

$$\text{So } {\cdot}8274 \times 100 = 82{\cdot}74\;;\quad 192{\cdot}6 \div 100 = 1{\cdot}926$$

$$ {\cdot}0245 \times 1000 = 24{\cdot}5\;;\quad {\cdot}8 \div 1000 = {\cdot}0008$$

133. To write tenths, hundredths, thousandths, etc., as decimals.

If the numerator has as many figures as there are 0's in the denominator, simply write the numerator with a point to the left of it.

If the numerator has fewer figures than the number of 0's in the denominator, supply the deficiency by 0's, and write a point to the left of the whole.

An improper fraction must be reduced to a mixed number.

EXAMPLES. $\frac{43}{100} = {\cdot}43$, but $\frac{43}{1000} = {\cdot}043$

$\frac{8}{10} = {\cdot}8$, but $\frac{8}{100} = {\cdot}08$

$\frac{17}{10} = 1\frac{7}{10} = 1{\cdot}7$, $\frac{1357}{100} = 13\frac{57}{100} = 13{\cdot}57$

134. To convert a decimal fraction to a vulgar fraction.

Take the decimal for the numerator and 1 followed by as many ciphers as there are figures after the decimal point for the denominator.

EXAMPLES. $\cdot 7 = \frac{7}{10}$; $\cdot 15 = \frac{15}{100}$; $\cdot 08 = \frac{08}{100}$ or $\frac{8}{100}$; $23.205 = 23\frac{205}{1000}$.

EXERCISE XLV.

1. Express as decimal fractions, $\frac{3}{10}$, $\frac{47}{100}$, $\frac{8}{100}$, $\frac{16}{1000}$, $\frac{508}{1000}$.

2. Express as vulgar fractions, ·7, ·63, ·04, ·017, ·009, ·706.

3. Express decimally, $7\frac{8}{10}$, $61\frac{35}{100}$, $600\frac{6}{100}$, $\frac{23}{10000}$, $\frac{86}{10}$, $\frac{965}{100}$.

4. Express as mixed numbers, 5·6, 24·01, 40·4, 60·06, 2·3456, 77·77.

5. Express decimally, six tenths, seventy-eight hundredths, nine thousandths, twenty-five tenths, eighteen ten-thousandths, four hundred and five millionths.

6. Multiply 3·14159 by 10, 100, and 1000 respectively.

7. Divide 17·308 by 10, 100, and 1000 respectively.

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ADDITION OF DECIMALS.

135. Decimals are added in the same way as whole numbers. We must take care to place units under units, tens under tens, tenths under tenths, and so on. This is easily done by keeping the decimal points under one another. We then add each column, beginning at the right, and carry as in adding whole numbers. Place the decimal point in the result under the decimal point above.

Example 1. Add together 17·82, 4·087, 63·8, and ·75.

17·82
4·087
63·8
·75
86·457

By vulgar fractions,

$$
\begin{aligned}
& 17\tfrac{82}{100} + 4\tfrac{87}{1000} + 63\tfrac{8}{10} + \tfrac{75}{100} \\
&= 17 + 4 + 63 + \tfrac{82}{100} + \tfrac{87}{1000} + \tfrac{8}{10} + \tfrac{75}{100} \\
&= 84 + \frac{820 + 87 + 800 + 750}{1000} \\
&= 84 + \tfrac{2457}{1000} = 84 + 2\tfrac{457}{1000} \\
&= 86\tfrac{457}{1000}
\end{aligned}
$$

EXERCISE XLVI.

Add

1.
3·87
49·6
·965
8·7
·538

2.
95·6
8·165
347·85
29·478
6·9

3.
1·5368
·086
46·3
5·478
165·9

4.
785·6
28·74
8·568
·3579
45·

5.
8·9
47·65
376·583
98·
4·67

6.
·08
·008
·0008
·8
8·

Find the result of

7. 13·65 + ·9 + ·068 + 45·7 + 76·865.

8. ·7 + ·08 + ·009 + 4 + 50 + 600.

9. 3·5 + 72·83 + 164·782 + 365·9 + 64.

10. 6·1875 + 48·568 + ·9306 + 8·8 + 95·6.

11. 8·8 + 80·08 + 800·008 + 8000·0008 + 80000·00008.

12. 777·7 + 77·77 + 7·777 + ·7777 + ·07777.

13. Find the sum of ·5, ·75, ·375, ·1875, and ·09375, and verify the result by vulgar fractions.

14. Prove that the sum of 4·13, ·068, 52·4, ·86, and 9·542 is an integer.

SUBTRACTION OF DECIMALS.

136. The operation of subtraction in decimals is performed in the same manner as in whole numbers. We must take care to place the less number under the greater, so that units may be under units, tens under tens, tenths under tenths, etc.

If the upper number has fewer decimal places than the lower, ciphers may be affixed to it.

Place the decimal point in the result under the decimal point above.

Example 1. From 37·6 subtract 8·45.

$$\begin{array}{r} 37{\cdot}60 \\ 8{\cdot}45 \\ \hline 29{\cdot}15 \end{array}$$

By vulgar fractions,

$$\begin{aligned} & 37\tfrac{6}{10} - 8\tfrac{45}{100} \\ &= 37 - 8 + \tfrac{6}{10} - \tfrac{45}{100} \\ &= 29 + \frac{60 - 45}{100} = 29\tfrac{15}{100} \end{aligned}$$

Example 2. Find the difference between ·396874 and ·85.

$$\begin{array}{r} {\cdot}850000 \\ {\cdot}396874 \\ \hline {\cdot}453126 \end{array}$$

Example 3. *A* owns ·175 of a farm, *B* owns ·25, and *C* owns the rest. *C's* share is worth \$165 more than the shares of *A* and *B* together. Find the value of the property.

$$\begin{array}{rr} A\text{'s share} = & {\cdot}175 \\ B\text{'s} \quad \text{“} \quad = & {\cdot}25 \\ \hline & {\cdot}425 \end{array} \qquad \begin{array}{rr} & 1{\cdot}000 \\ & {\cdot}425 \\ \hline C\text{'s} = & {\cdot}575 \end{array}$$

$$\begin{array}{r} {\cdot}575 \\ {\cdot}425 \\ \hline {\cdot}050 \end{array}$$

C's share exceeds the sum of *A* and *B's* by .05 or $\frac{5}{100}$.

$\frac{5}{100}$ths = \$165
$\frac{1}{100}$ths = \$35
whole = \$3500.

EXERCISE XLVII.

1.	From 18·369 Take 9·183	2.	1·514 ·8728	3.	48·6 37·564
4.	·103 ·002875	5.	13. 5·1768	6.	·08 ·00008

Find the result of :

7. 478·3 – 96·47
8. 15·68 – 9·896
9. ·135 – ·0579
10. 3·465 – 2·68
11. 158·7 – 81·84
12. 1 – ·99999

Find the difference between, and verify each result by vulgar fractions :

13. 6.25 and 8.75
14. ·175 and ·08
15. 3·51 and 1·687
16. 66·6 and 6·66

17. How much must be added to the sum of ·1234, 1·234, 12·34 and 123·4 to make 150 ?

18. How much does the sum ·5, ·25, ·125, ·0625 and ·03125 fall short of unity ?

19. How much nearer to 1·35768 is 1·36 than 1·35 ?

20. *A* did ·18 of a certain work, *B* did ·23, *C* did ·3 ; how much remained to be done to finish it ?

21. If ·312 of a property be worth $240 more than ·275 of it, what is the value of the property ?

22. *A* owns ·465 of a farm and *B* owns the rest. *B's* share is worth $350 more than *A's*. Find the value of each man's share.

MULTIPLICATION OF DECIMALS.

137. To multiply a decimal by a whole number.

EXAMPLE 1. ·3 × 2. $\frac{3}{10} \times 2 = \frac{6}{10}$: ∴ ·3 × 2 = 6.

EXAMPLE 2. ·06 × 7. $\frac{6}{100} \times 7 = \frac{42}{100}$: ∴ ·6 × 7 = ·42.

EXAMPLE 3. ·012 × 6. $\frac{12}{1000} \times 6 = \frac{72}{1000}$: ∴ ·012 × 6 = ·072.

EXAMPLE 4. 3·645 × 47. $\frac{3645}{1000} \times 47 = \frac{3645 \times 47}{1000} = \frac{171315}{1000}$ = 171·315 : ∴

$$\begin{array}{r} 3{\cdot}645 \\ 47 \\ \hline 25515 \\ 14580 \\ \hline 171{\cdot}315 \end{array}$$

Rule. Multiply as in whole numbers, and place the decimal point so that the product shall have as many figures after the point as there are in the multiplicand.

138. To multiply a decimal by a decimal.

EXAMPLE 1. ·4 × ·2. $\frac{4}{10} \times \frac{2}{10} = \frac{8}{100}$: ∴ ·4 × ·2 = ·08.

EXAMPLE 2. ·03 × ·6. $\frac{3}{100} \times \frac{6}{10} = \frac{18}{1000}$: ∴ ·03 × ·6 = ·1018.

EXAMPLE 3. ·007 × ·08. $\frac{7}{1000} \times \frac{8}{100} = \frac{56}{100000}$: ∴ ·007 × ·08 = ·00056.

Example 4. $56{\cdot}328 \times 4{\cdot}63.$ $\frac{56328}{1000} \times \frac{463}{100} = \frac{56328 \times 463}{100000} = \frac{26079864}{100000} = 260{\cdot}79864$: ∴

$$\begin{array}{r} 56{\cdot}328 \\ 463 \\ \hline 168984 \\ 337968 \\ 225312 \\ \hline 260{\cdot}79864 \end{array}$$

Rule. Multiply as in whole numbers, and place the decimal point in the product so that the number of decimal places (*i. e.*, figures after the decimal point), shall be the sum of the numbers of decimal places in the multiplicand and multiplier.

Note. If the number of figures in the product be less than the above sum, ciphers must be prefixed to the left of the product.

Example 1. Find the value of 4·37 pounds at $1.75 a pound.

$$\begin{array}{r} 1{\cdot}75 \\ 4{\cdot}37 \\ \hline 1225 \\ 525 \\ 700 \\ \hline 7{\cdot}6475 \end{array} \quad \text{Result \$7·6475.}$$

Example 2. A, who owns ·8 of a property, sells ·6 of his share to B. What part of the property does B get, and how much does A retain?

B gets ·8 of ·6 = ·48

A had ·8 × 1 ; he sells ·8 × ·6 ; 1 − ·6 = 4

∴ A retains ·8 × ·4 = ·32

Verification: ·48 + ·32 = ·8

EXERCISE XLVIII.

Find the result and verify by vulgar fractions :

1. ·3 × 5
2. ·4 × ·5
3. ·6 × ·02
4. 1·6 × 4
5. ·08 × ·03
6. 13·5 × ·12
7. 3·125 × 8
8. ·13 × ·004
9. 268· × ·23
10. 7·164 × 60
11. .2 × ·3 × ·4
12. ·87 × 6·9
13. ·1267 × 500
14. 2·608 × ·036
15. 80·57 × 24·8
16. ·0516 × ·078

17. Find the value of 4·2 pounds at $1.75 a pound.

18. Find the cost of ·3125 of a yard at $2.40 a yard.

19. A, who owns ·7 of a farm, sells ·65 of his share to B. What part of the farm does B get, and what part has A remaining.

20. A does ·35 of a certain work ; B does ·6 of the remainder, and then C finishes it. What part of the whole work is done by C ?

21. Find the value of 105·84 acres at $24.65 an acre.

◆

DIVISION OF DECIMALS.

139. To divide a decimal by a whole number.

Example 1. ·6 ÷ 2. $\frac{6}{10} \div 2 = \frac{3}{10}$: ∴ ·6 ÷ 2 = ·3.

Example 2. ·15 ÷ 5. $\frac{15}{100} \div 5 = \frac{3}{100}$: ∴ ·15 ÷ 5 = ·03.

Example 3. ·3 ÷ 4. $\frac{3}{10} \div 4 = \frac{300}{1000} \div 4 = \frac{75}{1000}$: ∴ ·3 ÷ 4 = ·300 ÷ 4 = ·075.

Example 4. 17·36 ÷ 8. $\frac{1736}{100} \div 8 = \frac{217}{100}$ = 2·17 : ∴ 17·36 ÷ 8 = 2·17.

Example 5. 8 3164 ÷ 34 $\frac{83164}{10000} \div 34 = \frac{2446}{10000} = 2446$,

```
or       34)8.3164(·2446
              68
              ---
              151
              136
              ---
               156
               136
               ---
                204
                204
```

Rule. Divide as in whole numbers, and point off as many decimal places in the quotient as there are in the dividend.

Note. Ciphers affixed to the dividend must be counted.

140. To divide a decimal by a decimal.

Example 1. ·6 ÷ ·3. $\frac{6}{10} \div \frac{3}{10} = \frac{6}{10} \times \frac{10}{3} = \frac{6}{3}$ ∴ ·6 ÷ ·3 = 6 ÷ 3 = 2.

Example 2. ·012 ÷ ·04. $\frac{12}{1000} \div \frac{4}{100} = \frac{12}{1000} \times \frac{100}{4} = \frac{12}{10} \times \frac{1}{4} = \frac{1}{4}$ of $\frac{12}{10}$. Hence ·012 ÷ ·04 = 1·2 ÷ 4 = ·3.

Example 3. 17·1348 ÷ 1 2. $\frac{171348}{10000} \div \frac{12}{10} = \frac{171348}{10000} \times \frac{10}{12} = \frac{171348}{1000} \times \frac{1}{12} = \frac{1}{12}$ of $\frac{171348}{1000} = \frac{1}{12}$ of 171·348. Hence 17·1348 ÷ 1·2 = 171·348 = 14·279.

Example 4. 13·8105 ÷ ·165. $\frac{138105}{10000} \div \frac{165}{1000} = \frac{138105}{10000} \times \frac{1000}{165} = \frac{138105}{10} \times \frac{1}{165} = \frac{1}{165}$ of 13810·5. Hence 13·8105 ÷ ·165 = 13810·5 ÷ 165.

```
165)13810.5(83·7
     1320
     ----
      610
      495
      ---
       155
       155
```

Rule. Make the divisor a whole number by moving the decimal point to the right of the last figure, and move the point in the dividend the same number of places to the right as it has been moved in the divisor. Divide as in whole numbers, and point off in the quotient as many decimal places as there are now in the dividend.

Example. Divide 6·5 by 4·16.

```
416·)650·(1·5625
     416
     ----
     2340
     2080
     -----
      2600
      2496
      -----
       1040
        832
       -----
        2080
        2080
```

Here we count the 4 ciphers supplied to the dividend to complete the division. ∴ the number of places after the decimal point in the quotient is 4.

Example 2. Divide 52·7 by ·017.

```
17)52700(3100.
   51
   ---
    17
    17
   ---
     00
```

Example 3. If 45·6 pounds of tea be worth $29.64, find the cost per pound.

```
456)296·4(·65
    273·6
    -----
     2280
     2280
     ----    Result = $·65 = 65c.
```

Example 4. If ·45 of a yard be worth \$1.53, what will be the value of ·75 of a yard?

$$\text{Value of } \cdot45 \text{ yard} = \$1.53$$

$$\therefore \quad \text{“} \quad 1 \quad \text{“} \quad = \frac{1\cdot53}{\cdot45} = \frac{153}{45}$$

$$\therefore \quad \text{“} \quad \cdot75 \quad \text{“} \quad = \frac{\overset{51}{\cancel{153}} \times \overset{.05}{\cancel{75}}}{\underset{3}{\cancel{45}}} = 51 \times \cdot05 = \$2.55.$$

EXERCISE XLVIX.

Find the result and verify by vulgar fractions :

1. ·24 ÷ 6
2. 3·2 ÷ 8
3. ·216 ÷ 9
4. 73·8 ÷ 12
5. 1·17 ÷ ·9
6. 7·5 ÷ ·15
7. ·324 ÷ 3·6
8. 16·5 ÷ 4·4
9. ·6 ÷ ·125
10. 9·855 ÷ 7·3
11. 98·55 ÷ ·73
12. 985·5 ÷ ·073
13. 552·5325 ÷ ·325.
14. 5·525325 ÷ 32·5
15. 590·4825 ÷ ·03275
16. 5·904825 ÷ 18·05

17. If 7·6 pounds of tea cost \$5.70, find the value of 1 pound.

18. If a man does ·36 of a work in 4·5 hours, in what time can he do the remainder?

19. In a certain school ·35 of the pupils are in the 1st class, ·28 in the 2nd, ·22 in the 3rd, and the remaining 18 in the 1st class. How many pupils are there in the school?

20. A can saw ·16 of a cord of wood in 1 hour, B can saw ·18. In what time can they together saw 8·5 cords?

141. In practical calculations, quotients will seldom terminate, and the usual way is to obtain the quotient to a certain number of places after the decimal point, and then add 1 to the last figure of the quotient if the remainder is greater than half the divisor, if not to neglect it altogether.

Example 1. Divide 8·7 by ·98 to the nearest figure in the third place.

98)870·(8·877
784
860
784
760
686
740
686
24

Since the remainder is greater than half the divisor, we call the result 8·878.

Example 2. Divide ·526 by 6·8 to the nearest figure in the third place.

68)5·260(·077
476
500
476
24

In both of the above examples the division is carried on till three figures after the decimal point in the dividend have been brought down.

EXERCISE L.

Divide to the nearest figure in the third place :

1. 3·64 by ·67
2. ·654 by 8·6
3. 1·3 by 9·7
4. ·038 by ·74
5. ·637 by ·89
6. 26·7 by 9·3
7. 476·8 by 76·5
8. 7·038 by 60·54.
9. 317 by 67

10. The circumference of a circle is 3,1416 times the diameter ; find, to three places of decimals, the diameter of a circular course of 1 mile (5280 feet).

11. A metre is 39·37 inches ; find how many metres there are in 475·8 inches.

12. If 47·5 bushels of wheat be sold for $56.85, find the price per bushel to the nearest cent.

13. The weight of a pint of alcohol is ·794 of the weight of a pint of water. If a pint of water weighs 1·25 pounds, how much alcohol will weigh one pound ?

14. Find the result, to 3 places of decimals, of ·815 × ·00867.

142. To convert a vulgar fraction into a decimal.

$$\tfrac{1}{2} = \tfrac{1}{2} \text{ of } 1 = \tfrac{1}{2} \text{ of } 1{\cdot}9 = {\cdot}5 \text{ ; or } \begin{array}{r} 2)\underline{1{\cdot}0} \\ {\cdot}5 \end{array}$$

$$\tfrac{3}{4} = \tfrac{1}{4} \text{ of } 3 = \tfrac{1}{4} \text{ of } 3{\cdot}00 = {\cdot}75 \text{ ; or } \begin{array}{r} 4)\underline{3{\cdot}00} \\ {\cdot}75 \end{array}$$

$$\tfrac{5}{8} = \tfrac{1}{8} \text{ of } 5 = \tfrac{1}{8} \text{ of } 5{\cdot}000 = {\cdot}625 \text{ ; or } \begin{array}{r} 8)\underline{5{\cdot}000} \\ {\cdot}625 \end{array}$$

$$\tfrac{1}{3} = \tfrac{1}{3} \text{ of } 1 = \tfrac{1}{3} \text{ of } 1{\cdot}000 = {\cdot}333\tfrac{1}{3} \text{ ; or } \begin{array}{r} 3)\underline{1{\cdot}000\phantom{\tfrac{1}{3}}} \\ {\cdot}333\tfrac{1}{3} \end{array}$$

$$\tfrac{5}{6} = \tfrac{1}{6} \text{ of } 5 = \tfrac{1}{6} \text{ of } 5{\cdot}000 = {\cdot}833\tfrac{2}{3} \text{ ; or } \begin{array}{r} 6)\underline{5{\cdot}000\phantom{\tfrac{2}{3}}} \\ {\cdot}833\tfrac{2}{3} \end{array}$$

Rule. Divide the numerator by the denominator, according to the Rule for Division of Decimals.

143. Terminating decimals. Since, for every 0 we affix to the numerator, we multiply the dividend by 10, if the denominator contains no other prime factors than 2's and 5's (the factors of 10), then, by affixing a sufficient number of 0's to the numerator, we will at length get a number exactly divisible by the denominator, and the operation will terminate.

144. If the denominator contains any other prime factor than 2's and 5's, we cannot introduce such factor into the dividend by multiplying by 10 (5 × 2), and therefore we cannot make the dividend exactly divisible by the denominator.

In this case we cannot find a decimal exactly equal to the vulgar fraction, but we can carry on the operation till the remainder becomes so small that it may be omitted. For practical purposes, two or three places after the decimal point are usually enough.

Note. The vulgar fraction is supposed to be in its lowest terms.

EXAMPLE 1. Convert $\frac{7}{13}$ of a dollar into a decimal.

$$\begin{array}{r l}
13)7{\cdot}000({\cdot}538\tfrac{6}{13} & \text{Result }\$.538 \\
\underline{65} & \\
50 & \\
\underline{39} & \\
110 & \\
\underline{104} & \\
6 &
\end{array}$$

If we take ·538 as the result, we omit only $\frac{6}{13}$ of $\frac{1}{1000}$ of a dollar or $\frac{6}{13000}$.

EXAMPLE 2. Express $\frac{3}{11}$ of a foot decimally:

$$\tfrac{1}{11} \text{ of } 3{\cdot}0000 = {\cdot}2727\tfrac{2}{11} \text{ or } 11)\underline{3{\cdot}0000} \atop {\cdot}2727\tfrac{2}{11}.$$

If we take ·2727 as the result, we omit only $\frac{2}{11}$ of $\frac{1}{10000}$, or $\frac{2}{110000}$ of a foot; a very small quantity.

EXAMPLE 3. Express $\frac{5}{12}$ decimally.

$$\begin{array}{l}
12)\underline{5{\cdot}0000} \\
\quad{\cdot}4166\tfrac{2}{3} \quad \text{Result } {\cdot}4166\text{; or more nearly } {\cdot}4167.
\end{array}$$

145. Recurring decimals. In the last two examples we see that the same figure or figures continually recur in the same order. Such decimals are called Recurring, Repeating or Circulating Decimals : **Pure**, if the recurring part begins immediately after the decimal point; **Mixed**, if one or more figures, which do not recur, precede the recurring part.

The recurring part, called the Repetend or Period, is usually written only once, and a point is put over the first and last figure of the Repetend ; or if there is only one figure, a point is put over that figure.

$$\text{Thus } \tfrac{3}{11} = \cdot\dot{2}\dot{7}\,; \quad \tfrac{5}{12} = \cdot 41\dot{6}$$

146. Every vulgar fraction which does not produce a terminating decimal must give a recurring one, if the operation be carried far enough. Take $\frac{5}{7}$ as an example. The remainders must be all less than 7 ; ∴ we can have, at the most, only six different remainders ; ∴ after the sixth figure of the decimal, if not before, the 5, or one of the remainders, must recur, and then the quotient will recur.

$$\begin{array}{rl} & 7)7\cdot000000 \\ \hline & \cdot714285\tfrac{5}{7} \\ \text{Remainders} & 142645 \end{array}$$

It is plain that if the operation be continued the same figures will recur,

$$\therefore \text{ the result is } \cdot\dot{7}1428\dot{5}$$

Note. The numerator 5 being less than 7 is one of the possible remainders.

EXERCISE LI.

Express decimally :

1. $\frac{1}{8}$
2. $\frac{3}{8}$
3. $\frac{7}{8}$
4. $\frac{11}{16}$
5. $\frac{13}{20}$
6. $\frac{9}{40}$
7. $2\frac{3}{4}$
8. $7\frac{4}{5}$
9. $\frac{31}{32}$
10. $\frac{47}{25}$
11. $7\frac{7}{80}$
12. $\frac{3}{400}$

Express decimally to 4 places :

13. $\frac{2}{3}$ 14. $\frac{5}{6}$ 15. $\frac{6}{7}$ 16. $\frac{8}{11}$

17. $\frac{7}{12}$ 18. $\frac{13}{44}$ 19. $\frac{8}{17}$ 20. $\frac{25}{19}$

Express as recurring decimals :

21. $\frac{1}{9}$ 22. $\frac{5}{9}$ 23. $\frac{17}{99}$ 24. $61\frac{11}{90}$

25. $\frac{1}{990}$ 26. $\frac{57}{44}$ 27. $13\frac{4}{7}$ 28. $9\frac{13}{55}$

29. $6\frac{5}{13}$ 30. $8\frac{2}{37}$ 31. $\frac{51}{52}$ 32. $3\frac{4}{65}$

33. $47\frac{1}{185}$ 34. $1\frac{103}{148}$ 35. $5\frac{2}{27}$ 36. $\frac{1}{135}$

147. To convert a terminating decimal into a vulgar fraction in its lowest terms.

$$·05 = \frac{5}{100} = \frac{1}{20}. \qquad ·75 = \frac{75}{100} = \frac{15}{20} = \frac{3}{4}.$$

$$6·375 = 6\frac{375}{1000} = 6\frac{75}{200} = 6\frac{15}{40} = 6\frac{3}{8}.$$

Rule. Take for the numerator the figures of the decimal written as a whole number, and for the denominator 1 followed by as many 0's as there are figures after the decimal point. The fraction thus formed must be reduced to lowest terms.

Note. In some cases it is easier to reduce a fraction to lowest terms by first multiplying the numerator and denominator by 2, 4 or 8.

Thus, $\frac{375}{1000} = \frac{3000}{8000} = \frac{3}{8}$.

EXERCISE LII.

Convert to vulgar fractions in lowest terms :

1. ·25 2. 3·05 3. ·225 4. ·0175

5. 17·6 6. 9·875 7. ·0008 8. 7·65

9. ·064 10. 12·35 11. ·4375 12. 4·56

148. To find the vulgar fraction which produces a given pure recurring decimal.

Example 1. What vulgar fraction produces $.\dot{6}$?

$$\begin{array}{rcl} 10 \text{ times } \cdot\dot{6} &=& 6\cdot66666 \text{ etc.} \\ 1 \text{ time } \cdot\dot{6} &=& \cdot66666 \text{ etc.} \\ \therefore\ 9 \text{ times } \cdot\dot{6} &=& 6 \quad \therefore\ \cdot\dot{6} = \tfrac{6}{9} = \tfrac{2}{3}. \end{array}$$

Example 2. What vulgar fraction produces $\cdot\dot{5}\dot{4}$?

$$\begin{array}{rcl} 100 \text{ times } \cdot\dot{5}\dot{4} &=& 54\cdot5454 \text{ etc.} \\ 1 \text{ time } \cdot\dot{5}\dot{4} &=& \cdot5454 \text{ etc.} \\ \therefore\ 99 \text{ times } \cdot\dot{5}\dot{4} &=& 54 \quad \therefore\ \cdot\dot{5}\dot{4} = \tfrac{54}{99} = \tfrac{6}{11}. \end{array}$$

Rule. Take the recurring period, written as a whole number, for the numerator, and for the denominator as many 9's as there are figures in the period.

149. To find the vulgar fraction which produces a given mixed recurring decimal.

Example 1. What fraction produces $\cdot41\dot{6}$?

$$\begin{array}{rcl} 1000 \text{ times } \cdot41\dot{6} &=& 416\cdot66666, \text{ etc.} \\ 100 \quad \text{“} \quad \text{“} &=& 41\cdot66666, \text{ etc.} \\ \therefore\ 900 \quad \text{“} \quad \text{“} &=& 416 - 41. \end{array}$$

$$\therefore\ \cdot41\dot{6} = \frac{416 - 41}{900} = \tfrac{375}{900} = \tfrac{3000}{7200} = \tfrac{30}{72} = \tfrac{5}{12}.$$

Example 2. What fraction produces $\cdot3\dot{5}\dot{4}$?

$$\begin{array}{rcl} 1000 \text{ times } \cdot3\dot{5}\dot{4} &=& 354\cdot5454, \text{ etc.} \\ 10 \quad \text{“} \quad \text{“} &=& 3\cdot5454, \text{ etc.} \\ \therefore\ 990 \quad \text{“} \quad \text{“} &=& 354 - 3. \end{array}$$

$$\therefore\ \cdot3\dot{5}\dot{4} = \frac{354 - 3}{990} = \tfrac{351}{990} = \tfrac{117}{330} = \tfrac{29}{110},$$

Rule. To find the numerator, subtract from the given decimal, to the end of the first period, the part that does not recur, both being regarded as whole numbers; for the denominator write as many 9's as there are recurring figures, followed by as many 0's as there are non-recurring figures.

EXERCISE LIII.

Convert to vulgar fractions, in lowest terms, the following decimals:

1. $7\cdot\dot{6}$ 2. $5\cdot\dot{8}$ 3. $12\cdot\dot{9}$ 4. $5\cdot\dot{3}$

5. $6\cdot\dot{7}\dot{2}$ 6. $11\cdot1\dot{6}$ 7. $10\cdot08\dot{3}$ 8. $\cdot6\dot{8}\dot{1}$

9. $\cdot\dot{9}6\dot{2}$ 10. $8\cdot\dot{1}8\dot{5}$ 11. $\cdot4\dot{7}2\dot{9}$ 12. $\cdot1\dot{7}0\dot{2}$

13. $\cdot\dot{7}1428\dot{5}$ 14. $\cdot307692$ 15. $\cdot\dot{1}92307\dot{6}$ 16. $\cdot034\dot{0}\dot{9}$

17. How much more nearly is 3·1416 represented by $\frac{355}{113}$ than by $\frac{22}{7}$?

150. Non-terminating decimals are carried to any desired number of places and then treated as terminating decimals. In the case of the comparatively few decimals whose figures soon recur, we can, by carrying the decimals far enough, obtain the results of addition, subtraction, multiplication and division in a recurring form. But in some cases this result is more easily obtained by using vulgar fractions.

Example 1. Multiply $\cdot\dot{6}$ by $\cdot\dot{8}\dot{1}$.

$\cdot\dot{6} = \frac{6}{9} = \frac{2}{3}$

$\cdot\dot{8}\dot{1} = \frac{81}{99} = \frac{9}{11}$

$\frac{2}{3} \times \frac{9}{11} = \frac{6}{11} = \cdot\dot{5}\dot{4}$

or

```
     ·8181818
        ·6666
     --------
    4|9090908
   49|090908
  490|90908
 4909|0908
 ----|-------
·5454|
```

We get the result correctly to 4 places by carrying 2 from the 5th place; ∵ 19 is nearer to 20 than to 10.

Example 2. Express $\frac{6}{7}$ and $\frac{35}{12}$ decimally to 3 places after the decimal point, and find how much the product of the decimals differs from that of the given fractions.

$$7\overline{)6{\cdot}000} \qquad 12\overline{)35}$$
$$\;\;{\cdot}857 \qquad\qquad 2{\cdot}917$$

We increase the last figure by 1, ∴ the remainder is more than half the divisor.

$$\begin{array}{r} 2{\cdot}917 \\ {\cdot}857 \\ \hline 20419 \\ 14585 \\ 23336 \\ \hline 2{\cdot}499869 \end{array}$$

$$\tfrac{35}{12} \times \tfrac{6}{7} = \tfrac{5}{2} = 2{\cdot}5$$

$$\begin{array}{r} 2{\cdot}500000 \\ 2{\cdot}499869 \\ \hline \text{Difference} = {\cdot}000131 \end{array}$$

EXERCISE LIV.

Find the results of :

1. ${\cdot}\dot{6} + {\cdot}\dot{5}\dot{4} + {\cdot}0\dot{8}$ 2. ${\cdot}\dot{8} - {\cdot}\dot{1}8\dot{5}$

3. $1{\cdot}375 - {\cdot}\dot{9}6\dot{2}$ 4. ${\cdot}\dot{3} \times {\cdot}\dot{6}$

5. $1{\cdot}\dot{7}\dot{2} \times 3{\cdot}\dot{4}$ 6. $6{\cdot}\dot{7} \div 2{\cdot}\dot{6}$

7. ${\cdot}38\dot{6}\dot{3} \div 2{\cdot}\dot{2}9\dot{7}$ 8. $15{\cdot}41\dot{6} \div 1{\cdot}6\dot{8}\dot{1}$

9. ${\cdot}\dot{2}9\dot{6} \times {\cdot}\dot{4}\dot{5} \div 2{\cdot}\dot{7}4\dot{0}$. 10. ${\cdot}3\dot{7}\dot{1} \div 5$

Find the result to 5 decimal places of :

11. $1{\cdot}8\dot{5} \div 7{\cdot}65$ 12. ${\cdot}\dot{6}1\dot{7} \times {\cdot}03\dot{8}$

13. $63{\cdot}\dot{5} + 7{\cdot}\dot{0}\dot{6} + 16{\cdot}0\dot{9} + {\cdot}\dot{8}1\dot{5} + {\cdot}1\dot{3}5\dot{7}$

14. How much does ·667 differ from ${\cdot}\dot{6}$?

15. If I spend ${\cdot}\dot{2}$, ${\cdot}\dot{2}\dot{7}$ and ${\cdot}\dot{1}\dot{4}$ of my money, what part

will I have left? Also, if the remainder is 40c., how much did I spend?

16. A buys $\cdot\dot{4}\dot{5}$ of a property, B buys $\cdot\dot{6}$ of the remainder, and C buys the rest. What part does C purchase?

151. To express a decimal of a concrete quantity as a compound quantity.

Example 1. Find the value of £2·31875.

$$\begin{array}{r} 2.31875 \\ 20 \\ \hline 6\cdot37500 \\ 12 \\ \hline 4\cdot50000 \\ 4 \\ \hline 2\cdot00000 \end{array}$$

·31875 of £1 = ·31875 of 20*s.* = ·31875 × 20*s.* = 6·375*s.*; ·375 of 1*s.* = ·375 of 12*d.* = ·375 × 12*d.* = 4·5*d.*; ·5*d.* = ·5 × 4 *fr.* = 2 *fr.* = $\frac{1}{2}$d.

Result. £2 6*s.* 4$\frac{1}{2}$*d.*

Example 2. Find the value of $8\cdot\dot{4}\dot{5}$ lbs. Troy.

$$\begin{array}{r} 8\cdot454545 \\ 12 \\ \hline 5\cdot454540 \\ 20 \\ \hline 9\cdot090800 \\ 24 \\ \hline 363200 \\ 181600 \\ \hline 2\cdot179200 \end{array}$$

Or $8\cdot\dot{4}\dot{5} = 8\frac{45}{99} = 8\frac{5}{11}$.

$$\begin{array}{r|cccc} & \text{lb.} & \text{oz.} & \text{dwt.} & \text{gr.} \\ 11) & 5 & 0 & 0 & 0 \\ \hline & & 5 & 9 & 2\frac{2}{11} \end{array}$$

Result. 8 lbs. 5 oz. 9 dwt. $2\frac{2}{11}$ grs.

Result. 8 lbs. 5 oz. 9 dwt. 2·18 grs. nearly.

Example 3. Find the value of 7·84 acres at £6·8$\dot{3}$ an acre.

$$\begin{array}{r}
6{\cdot}83333 \\
7{\cdot}84 \\
\hline
2733332 \\
5466664 \\
4783331 \\
\hline
53{\cdot}5733072 \\
20 \\
\hline
11{\cdot}4661540 \\
12 \\
\hline
5.5938480
\end{array}$$

Or, $£6\frac{83-8}{90} \times 7\frac{84}{100}$

$= £6\frac{5}{6} \times 7\frac{21}{25}$

$= £\frac{41}{6} \times \frac{196}{25} = £\frac{41}{3} \times \frac{98}{25}$

$= £\frac{4018}{75} = £53$ 11*s.* $5\frac{3}{5}$*d.*

Result. £53 11*s.* 5·6*d.* nearly.

Hence :

Rule. Multiply the decimal fraction by the number of units of the next lower denomination contained in one of the higher. Continue this process till there is no fractional part or till the lowest denomination is obtained. This process is similar to reduction descending.

Or reduce the decimal fraction to a vulgar fraction, and then find the value of the vulgar fraction as in (125).

EXERCISE LV.

Find the value of

1. ·025 of $6.
2. ·575 of $2.40.
3. 6·315 of $5.
4. ·1875 of £1.
5. 7·4375 of £1.
6. 17·86 of £2.
7. ·0625 of £1 4*s.*
8. 30·24 of 1 acre.
9. ·94375 of 1 lb. Troy.
10. 8·7 of 17*s.* 6*d.*
11. ·876 of 1 day.
12. ·99609375 of 1lb. avoir.
13. ·$\dot{6}$ of £1.
14. 1·58$\dot{3}$ of £1.
15. 2·$\dot{1}\dot{8}$ of $16·50.
16. ·79$\dot{5}\dot{4}$ of 1 mile.

17. $5·84$ of \$6.75.

18. $·\dot{9}7\dot{2}$ of 6*s.* 2*d.*

19. $.87\dot{0}\dot{3}$ of 1 ac. 2 p.

20. $3·\dot{1}4\dot{8}$ of a cubic yard.

21. If .125 of an acre be taken from ·56 of an acre find the value of the remainder.

22. The Julian correction makes the the year 365·25 days, the Gregorian, 365·2425 days; express the difference in minutes and seconds.

23. Find the sum of ·62 of £1 10*s.*, ·85 of 13*s.* 4*d.*, and 9·6 of 1*s.* 3*d.*

24. Express $2.\dot{5}7142\dot{8}$ of a pound avoirdupois in Troy weight.

25. Find the value of 3·45 lbs. of tea at £·356 a pound.

26. Express in roods and perches $·7\dot{7}\dot{2}$ of ·583 acres.

152. To express one quantity as a Decimal of another of the same kind.

EXAMPLE 1. What decimal of £3 10*s.* is 17*s.* 6*d.*?

£3 10*s.*
20
—
70
12
—
840*d.*

17*s.* 6*d.*
12
—
210*d.*

If the unit, or whole, be divided into 840 equal parts, 210 of these parts are to be taken.

$\frac{210}{840} = \frac{1}{4} = ·25.$ *Result.*

EXAMPLE 2. Express £2 8*s.* 10½*d* as a decimal of £5.

2) 1·0
12)10·5
20) 8·875
5) 2·44375
·48875 *Result.*

$\frac{1}{2}d. = ·5d.;\ 10·5d. = \frac{10·5}{12}s. = ·875s.;$

$8·875s. = \frac{8·875}{20}$ of £1

$\doteq ·44375$ of $\frac{1}{5}$ of £5 $= \frac{·44375}{5}$ of £5.

Hence:

Rule. Reduce both quantities to whole numbers of units of the same name. Form a vulgar fraction by taking the number in the quantity which is to be a decimal of the other for numerator and the number in the other quantity for denominator. Reduce the vulgar fsaction to a decimal.

Note. The method used in Example 2, which is similar to reduction ascending, is more convenient when a decimal of a simple quantity is required.

EXERCISE LVI.

1. What decimal of £1 is 8*s.* 9*d.*?

2. Express $3.75 as a decimal of $5.

3. Express 17 cwt. 3 qrs. 18 lbs. as a decimal of 1 ton.

4. What decimal of £2 10*s.* is 18*s.* 9*d.*?

5. What decimal of 1 lb. Troy is 7 oz. 13 dwt. 18 grs.?

6. Express 5 fur. 27 p. as a decimal of a mile.

7. What decimal of £6 12*s.* is 13*s.* 9*d.*?

8. Express 13*s.* 4*d.* as a decimal of £1.

9. Express £1 12*s.* 6*d.* as a decimal of £3 6*s.*

10. What decimal of a square foot is the size of a slate 9 in. broad, 15 in. long?

11. Express 1 oz. avoirdupois as a decimal of 1 oz. Troy.

12. What decimal of an hour is 42 m. 45 sec.?

13. What decimal of an acre is the area of a plot 165 ft. by 132 ft.?

14. A, B and C join in buying a boat, A contributing $16.50, B $22.50 and C the remainder. Express each man's share of the boat decimally.

15. A piece of squared timber is 12 ft 3 in. long, 2 ft. 6 in. wide and 2 ft. $1\frac{1}{2}$ in. thick. Express its bulk decimally in cubic feet.

16. From a sheet of paper 14 inches long, 9 inches broad a strip is cut off 1 inch wide all round : what decimal fraction of the sheet remains?

17. A gramme is 15·32 grs. Express a pound avoirdupois and a pound Troy respectively in grammes.

18. A metre is 39.37 inches. Express a mile decimally in kilometres. Also, express a kilometre decimally in furlongs.

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Miscellaneous Examples.

PAPER I.

1. How do you multiply or divide a decimal by 10, 100, &c. Prove the truth of your answer.

2. Divide ·0169 by ·013, 49·7 by ·0025 and ·00003 by ·001.

3. Define the terms fraction, denominator and numerator; and show that a fraction is multiplied if its denominator is divided, and divided if its denominator is multiplied.

4. Simplify the following fractional expressions :

$$(a)\ 5\tfrac{2}{3} \text{ of } \tfrac{1}{15} \div \left(6\tfrac{1}{5} - \tfrac{2}{25}\right). \quad (b)\ \tfrac{501}{1837}.$$

5. Find the product of ·47 and ·00432 ; and find the quotient of 4·8 by ·0016.

6. Reduce to decimals,

$$(a)\ \tfrac{3}{10} + \tfrac{7}{10000} + \tfrac{12}{10000000}\,; \ (b)\ \tfrac{29}{65}\,;$$

and find the vulgar fractions equivalent to ·2625 and $\cdot 602\dot{2}\dot{7}$.

PAPER II.

1. Assuming that if 6715100 be divided by 5068, the quotient is 1325; write down the quotient

(*a*) of 67151 by 13250,
(*b*) of ·06715100 by ·001325.

2. Shew that $2{\cdot}\dot{4}\dot{5} \times 3{\cdot}1\dot{6} \div {\cdot}01\dot{2}\dot{6}$ is equal to 615·6.

3. Simplify

$$\tfrac{18}{17}\left(1 - \tfrac{64}{81}\right) + \tfrac{8}{11} \times \tfrac{1}{6} \times \left(\tfrac{1}{2} + \tfrac{5}{12}\right) + \frac{1\frac{1}{2} \text{ of } 9\frac{1}{3}}{2\frac{3}{4} \text{ of } 7\frac{7}{11}}.$$

4. Simplify $\dfrac{{\cdot}875 \times {\cdot}\dot{2}7\dot{0}}{{\cdot}125 + {\cdot}125\dot{6}7\dot{5}} + \tfrac{3}{53}$.

5. To the summit of a tower there are 108 steps, each $\cdot 58\dot{3}$ feet high. What is the height of the tower?

6. On the equator the length of a pendulum beating seconds is 39.0168 inches, and at the poles 39.217 inches. Express the difference as the decimal of a yard.

PAPER III.

1. By what must ·000471 be divided to give 200 as the quotient?

2. Reduce $\frac{3}{800}$ and $\frac{5}{28}$ to decimals; and $\cdot 236\dot{1}$ to a vulgar fraction.

3. Find the greatest number of which 33495, 106260 are multiples; and the least number of which 26, 33, 39 and 44 are divisors.

4. A person who paid away two-thirds of the contents of his purse, and was robbed of two-sevenths of the remainder, found that he still had left $18.00. How much had he at first?

5. How many hours a day must 24 men work to finish as much in 5 days as 25 men could do in 4 days if they worked 6 hours a day?

6. A French metre is 39·37 inches nearly. Express a foot as a decimal of a metre correct to a tenth of an inch.

PAPER IV.

1. Divide ·005868 by ·036, and arrange the divisor, dividend and quotient in order of magnitude.

2. Add together $\frac{3}{4}$, $\frac{13}{16}$, $\frac{13}{32}$ and $\frac{7}{480}$; and divide the sum by the difference of $\frac{11}{24}$ and $\frac{7}{40}$.

3. Make out a bill for the following purchases:

10 lbs. of sugar at 10 cents a lb.
6 lbs. of tea at 82 cents a lb.
8 lbs. of coffee at 32 cents a lb.
12 lbs. of currants at $11\frac{1}{4}$ cents a lb.
10 lbs. of rice at 9 cents a lb.
9 lbs. of cheese at 18 cents a lb.

4. An express train runs 40 miles an hour, and an ordinary train 30 miles an hour; the express fare is $\frac{1}{2}$ cent a mile more than the ordinary. Find how much an hour a man's time is worth, if it cost him the same to travel by the one as by the other.

5. A lady buys at a shop $18\frac{1}{2}$ yards of silk at $1.80 a yard; 21 yards of calico at 23 cents a yard; 2 dozen pairs of gloves at 66 cents a pair; and 10 yards of ribbon at 44 cents a yard. What is the amount of her bill?

PAPER V.

1. How does it affect the value of a decimal to shift the decimal point two figures, (1) to the right, (2) to the left? Express as vulgar fractions in their lowest terms, ·75, ·0005, and as a decimal 17359 divided by one million.

2. What change in the decimal point will increase the value of a decimal one hundred fold? Express as a decimal the ten-thousandth part of ·7.

3. If the true value of a decimal be 3·14159, show that 3·142 is nearer the true value than 3·141.

4. Find a decimal which shall be within $\frac{1}{10000}$ of $\frac{17}{42}$.

5. Multiply 8·4 by ·009; and divide the product by 29·4 to five places of decimals.

6. If two horses are worth as much as 5 oxen, and 3 oxen as much as 16 sheep, what will the value of a horse be when the price of a sheep is $9?

7. A railway train moving at the uniform rate of 40 feet per second, leaves Toronto at 7 o'clock in the morning. When will it reach Guelph, the distance from Toronto to Guelph being 48½ miles?

SECTION VIII.

PRACTICE.

153. Practice is a neat way of multiplying by a fraction, which is used in computing the value of any number of articles at a given price, when there is a fraction in the price, or in the number of articles, or in both.

Example 1. Find the value of 3064 bushels of barley at $.85 a bushel.

$.50	½	$ 3064	=	value at	$1		a bushel.
$.25	½	1532	=	" "	$ .50		"
$.10	⅕	766	=	" "	$ .25		"
		306.40	=	" "	$ .10		"
		$2604.40	=	" "	$ .85		"

Example 2. Find the cost of 418 tons of iron at £5 8*s*. 9*d*. at ton.

		£		
5*s*.	¼	418	= cost at £1	a ton.
		5		
		2090	= “ “ £5	“
2/6	½	104 10	= “ “ 5*s*.	“
1/3	½	52 5	= “ “ 2*s*. 6*d*.	“
		26 2 6	= “ “ 1*s*. 3*d*.	“
		£2272 17*s*. 6*d*.	= “ “ £5 8*s*. 9*d*.	“

Example 3. If a ship be worth $7526 find the value of $\frac{5}{8}$ of the ship.

		$	
$\frac{4}{8}$	$\frac{1}{2}$	7526	= value of $\frac{8}{8}$
$\frac{1}{8}$	$\frac{1}{4}$	3763	= “ “ $\frac{4}{8}$
		940.75	= “ “ $\frac{1}{8}$
		$4703.75	= “ “ $\frac{5}{8}$

Example 4. Find the value of $\frac{19}{20}$ of $715.

	$	
$\frac{1}{20}$	715	= value of $\frac{20}{20}$
	35.75	= “ “ $\frac{1}{20}$
	$679.25	= “ “ $\frac{19}{20}$

Example 5. Find the value of 12 ac. 1 r. 23 p. at $5.80 an acre.

		$	
1 r.	$\frac{1}{4}$	5.80	= value of 1 ac.
		12	
		69.60	= “ “ 12ac.
20 p.	$\frac{1}{2}$	1.45	= “ “ 1r.
2 p.	$\frac{1}{10}$	.725	= “ “ 20p.
1 p.	$\frac{1}{2}$	.0725	= “ “ 2p.
		.03625	= “ “ 1p.
		$71.88375	= “ “ 12 ac. 1 r. 23p.

Note. 1 r. 23 p. = $\frac{63}{160}$ acre. Instead of finding the value of $\frac{63}{160}$ by dividing \$5.80 by 160 and multiplying the quotient by 63, we take for $\frac{40}{160}$, $\frac{1}{4}$ of \$5.80; for $\frac{20}{160}$, $\frac{1}{2}$ value of $\frac{40}{160}$; for $\frac{2}{160}$, $\frac{1}{10}$ value of $\frac{20}{160}$; and for the remaining $\frac{1}{160}$, $\frac{1}{2}$ value of $\frac{2}{160}$. Thus we get the value of

$$\frac{40 + 20 + 2 + 1}{160}.$$

EXAMPLE 6. Find the cost of 15 lbs. 6 oz. 14 dwt. 18 grs. of silver at 7*s.* 5*d.* an ounce.

			s.	*d.*
10 dwt.	$\frac{1}{2}$		7	5
4 "	$\frac{1}{5}$			186
		68	19	6
			3	8$\frac{1}{2}$
16 gr.	$\frac{1}{6}$		1	5$\frac{4}{5}$
2 gr.	$\frac{1}{8}$			2$\frac{29}{30}$
				$\frac{89}{240}$
		£69	4	11$\frac{51}{80}$

Instead of multiplying 7*s.* 5*d.* by 186, we might have also found the value by Practice, thus:

		£			
6/8	$\frac{1}{3}$	186			=value at £1
8*d.*	$\frac{1}{10}$	62			= " " 6/8
1*d.*	$\frac{1}{8}$	6	4		= " " 8*d.*
			15	6	= " " 1*d.*
		68	19	6	= " " 7*s.* 6*d.*

When done in the latter way, this is an example of what is called Compound Practice.

EXERCISE LVII.

Find the value of

1. $\frac{5}{12}$ of \$158.46.
2. $\frac{13}{16}$ of \$3184.
3. $\frac{3}{8}$ of £27 11*s.* 6*d.*
4. $\frac{7}{12}$ of £13 7*s.* 9*d.*
5. $\frac{14}{15}$ of \$6195.
6. $\frac{9}{10}$ of £37 16*s.* 8*d.*
7. 518 barrels of flour, at \$4.75 a barrel.
8. 1764 bushels of barley, at \$.65 a bushel.
9. 417 acres, at £2 11*s.* 6*d.* an acre.
10. 546 yards, at 17*s.* 6*d.* a yard.

11. 1372 sheep, at £1 18*s.* 10½*d.* each.

12. 35 ac. 3 r. 22 p., at $4.80 an acre.

13. 2 lbs. 6 oz. 12 dwt. 15 grs. of silver, at $1.92 per ounce.

14. 108 m. 5 f. 27 p. of railway, at $32,756 per mile.

15. £19 14*s.* 8*d.*, if £1 = $4.86.

16. £45 16*s.* 7½*d.*, if £1 = $4.86.

17. If a bankrupt pay 36 cents in the dollar, how much will a creditor receive on a debt of $734.80?

18. If a bankrupt pay only 2*s.* 9*d.* in the pound, how much will a creditor lose on a debt of £156 15*s.*?

19. If a train runs uniformly at 45 miles an hour, how far will it go in 4 h. 35 min. 45 sec.?

20. Find the value of an ingot of gold weighing 3 lbs. 7 oz. 17 dwts. 18 grs., at $20.70 an ounce.

21. Find the value of a farm containing 115 ac. 3 r. 17 p., at £5 11*s.* 8*d.* an acre.

22. Find the value of 4712$\frac{7}{12}$ yards of velvet, at £2 13*s.* 7½*d.* a yard.

23. If £1 = $4.86⅔, find the value of £135 3*s.* 9*d.*

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SECTION IX.

154. The average of two or more numbers is found by adding the numbers together and dividing their sum by the number of them.

10 acres yielded 170 bushels of wheat in 1879, 20 acres yielded 240 bushels in 1880, 12 acres yielded 360 bushels

in 1881, 35 acres yielded 770 bushels in 1882. Find the average yield per acre for these years.

The total number of acres is 77 ; the sum of the number of bushels is 1540 bushels : $1540 \div 77 = 20$.

Therefore the average yield per acre is 20 bushels.

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155. Per Centum, or per cent., means for a hundred : thus 7 per cent. means 7 for a hundred.

The 100 we refer to may be 100 of anything, viz. : $100, 100 oz., 100 acres, &c. And the number giving the per cent. is so many units of the same kind ; but we generally consider the 100 an abstract number.

Therefore, 4 per cent. of a quantity means 4 parts for a hundred of the quantity, or $\frac{4}{100}$ of the quantity.

Per cent. is frequently written % : thus 6 per cent. is written 6%, &c.

Example 1. How much is 45 % of 130 bushels of wheat?

$$45\ \%\text{ is }\tfrac{45}{100} = \tfrac{9}{20}\,;$$
$$\begin{aligned}\text{therefore } 45\ \%\text{ of }130\text{ bushels} &= \tfrac{9}{20}\text{ of }130\text{ bushels}\\ &= 58{\cdot}5\text{ bushels.}\end{aligned}$$

Example 2. The attendance of boys at a certain school is 350, 30 per cent. leave during each year : how many return at the beginning of a year?

$$30\text{ per cent. is }\tfrac{30}{100} = \tfrac{3}{10}\,;\ \text{therefore }\tfrac{7}{10}\text{ is left ;}$$
$$\therefore\ \text{number returning} = \tfrac{7}{10}\text{ of }350 = 245.$$

156. Commission is the charge made by an agent for buying or selling goods, and is usually a per centage on the value of the goods bought or sold.

Example 3. A land agent has placed at his disposal $800 to buy land in Manitoba ; allowing $2\frac{1}{2}$ % for commission, how much would his charge be?

$$2\tfrac{1}{2}\ \% \text{ is } \frac{2\frac{1}{2}}{100} = \tfrac{5}{200} = \tfrac{1}{40}\,;$$

$$\therefore \text{ Commission } = \tfrac{1}{40} \text{ of } \$800 = \$20.$$

Or

$$\begin{aligned} &\text{Commission on } \$100 &&= \$2\tfrac{1}{2}, \\ &\text{“} \qquad\quad \text{“} \quad \$1 &&= \$\tfrac{5}{200}\,; \\ \therefore\ &\text{“} \qquad\quad \text{“} \quad \$800 &&= \$\tfrac{5}{200} \text{ of } 800. \\ & &&= \$20. \end{aligned}$$

157. Brokerage is the charge made by a broker (agent) for buying or selling stocks, bills of exchange, goods, &c.

EXAMPLE 4. A broker has remitted to him \$750 with instructions to buy a bill of exchange, with the balance, after deducting his own charge of $\frac{1}{8}$ % brokerage. Find the amount of the bill.

$$\begin{aligned} &\$100 + \tfrac{1}{8}\ \% = \$100\tfrac{1}{8} \\ &\qquad \$100\tfrac{1}{8} \text{ buys } \$100 \text{ bill}\,; \\ \therefore &\qquad \$1 \text{ buys } \$\tfrac{800}{801} \text{ bill}\,; \\ \therefore &\qquad \$750 \text{ buys } \$\tfrac{800}{801} \times 750 \text{ bill} \\ \therefore &\qquad \text{Result} = \$749.06\tfrac{98}{267}. \end{aligned}$$

158. Insurance is a contract by which one party, on being paid a certain sum by another party, on property which is subject to risk, undertakes, in case of loss, to pay a specified sum to the owner of the property.

159. The sum paid by the insured is called the **Premium**; the **document** containing the contract is called the **Policy.**

EXAMPLE 5. Find the premium for insuring a house valued at \$6000 at $\frac{3}{4}$ %.

$$\tfrac{3}{4}\ \% \text{ is } \frac{\frac{3}{4}}{100} = \tfrac{3}{400}\,;$$

$$\therefore \text{ the premium } = \tfrac{3}{400} \text{ of } \$6000 = \$45.$$

Example 6. For what sum must a farmer insure his barn worth \$2500, so that in case of loss both the value of the barn and the premium may be recovered, rate of insurance being 3 per cent.

$$\$100 - \$3 = \$97\ ;\ \therefore\ \$100 \text{ insurance secures } \$97.$$

$$\begin{aligned} \text{Insurance on}\quad \$97 &= \$100 \\ \text{“}\quad\text{“}\quad \$1 &= \$\tfrac{100}{97} \\ \text{“}\quad\text{“}\quad \$2500 &= \$\tfrac{100}{97} \times 2500 = \$2577.31\tfrac{93}{97}. \end{aligned}$$

EXERCISE LVIII.

1. Find the premium on a house valued at \$2500, which is insured for 3 years at $1\frac{1}{4}$ per cent.

2. Find the commission on \$875 at $\frac{1}{2}$ %.

3. A produce dealer sends \$15000 to an agent to buy wheat; how many bushels of wheat can he buy, after deducting his commission of 5 per cent. on the value of the wheat to be bought, the price of wheat being \$1.15 per bushel?

4. A horse to be shown at the Toronto Exhibition (1882), worth \$1850 is insured at 5 per cent. so that if he be lost, both commission and value may be recovered: how much must be paid?

5. A broker charged $\frac{1}{8}$ % for transferring \$2000 from one account to another, how much did he get?

6. If you pay \$60 premium on \$3000, what is the rate per cent?

7. If \$42 be paid as insurance on a house for 3 years at $1\frac{1}{2}$ per cent.; what was the house valued at?

8. On a transaction of \$5500, a broker receives \$27.50: what did he charge on the \$100?

9. Spend \$15.00 during one week, \$12.00 during a second week, \$16.50 during a third, and \$17.50 during a fourth week. What will be the average sum spent each week?

160. Interest (Int.) is the sum of money paid for the use of money lent.

The money lent is called the **Principal**.

The interest on $100 for a year is called the **Rate per cent.**

The sum of the principal and the interest is called the **Amount**.

When interest is reckoned only on the principal or sum lent, it is **Simple Interest**.

When the interest, as soon as it becomes due, is added to the principal and forms a new principal for the next period, the interest is **Compound**.

EXAMPLE 1. Find the interest on $325 for one year at 7 %.

Interest at 1 % = $3.25
" 7 % = $22.75,
or, interest on $100 for 1 year = $7,
" $1 " = $\$\frac{7}{100}$
" $325 " = $\$\frac{7}{100} \times 325$
= $22.75.

EXAMPLE 2. Find interest on $325 for 3 years at 7 %. Also the amount

Interest on $100 for 1 year = $7
" " $1 " " = $\$\frac{7}{100}$
" " $1 " 3 years = $\$\frac{7 \times 3}{100}$ (3 times interest for 3 years)
" " $325 " " = $\frac{\$325 \times 7 \times 3}{100}$
= $68.25.
Amount = $325 + $68.25 = $393.25.

Hence the general rule for interest :

Rule. Multiply the principal by the rate per cent and by the number of years, or fraction of a year, and divide the product by 100.

EXAMPLE 3. Find the simple interest on $800 at 6½ % from 1st April, 1880, to 1st September, 1882.

April,	29
May,	31
June,	30
July,	31
Aug.,	31
Sept.,	1
	153

The day on which the money is borrowed is not included in the number of days but the day on which the money is paid is included, therefore we count only 29 days in April, but one in September. The whole time is 2 years and 153 days.

$$\begin{array}{llllllll} & \text{Interest} & \text{on} & \$100 & \text{for} & 365\ \text{days} & = & \$6\tfrac{1}{2} \\ & \text{"} & \text{"} & \$1 & \text{"} & 1\ \text{day} & = & \dfrac{\$6\frac{1}{2}}{100 \times 365} \\ \therefore & \text{"} & \text{"} & \$800 & \text{"} & 153\ \text{days} & = & \dfrac{\$800 \times 13 \times 153}{100 \times 730} \\ & & & & & & = & \$\ 21.80\ \text{nearly.} \end{array}$$

Interest on $800 at 6½ % for 2 years = 104

Interest on $800 at 6½ % for 2 years
and 153 days = $125.80

Note 1. When interest is required for days, as in the above example, instead of dividing by 365, double both the rate and days, and divide by 73000 ; thus shortening the work.

Note 2. Three days, called **Days of Grace**, are allowed after the time a bill or note is **nominally** due before it is **legally** due.

EXERCISE LIX.

1. Find the interest on $725, at 6 per cent., for one year.

2. What is the interest of $913, at 7 per cent., for one year?

3. Find the simple interest on $225.50, at 6½ per cent. per annum, for one year and six months.

4. Find the amount of $505.20, at 7 per cent. per annum for one year and 73 days, simple interest.

5. How much will $215.25 amount to at 8 per cent. per annum, for two years and a half, simple interest?

6. Find the simple interest, and the amount of $525.60, at 8 per cent. per annum, for 5 years, 215 days.

7. What is the simple interest on $220.30, at 6 per cent. per annum, for 3 years and 7 months?

8. Find the amount, and simple interest of $85.60, at 7 per cent. per annum, from the 12th January, 1882, to the 15th October, 1882.

9. A note for $110.50, at 7 per cent. per annum, dated the 3rd of May, 1882, is made for 3 months; what sum will pay it when due?

10. Find the simple interest on £212 10s. at £4 10s. per £100 per annum, for 2 years 35 days.

Example 4. If $500 amount to $605 in 3½ years, find the rate per cent. per annum.

$605 − $500 = $105 = interest.
Interest on $500 for 3½ years = $105
“ “ “ “ 1 year = $30.
“ “ $100 “ “ = $6.
Therefore the rate is 6 per cent.

Example 5. If $500 at 6 per cent. per annum amount to $605, find the time.

$\$605 - \$500 = \$105 =$ interest.

Interest on \$500 for 1 year at 6 per cent. = \$30.

Time to yield \$30 = 1 year,
“ “ “ $\$1 = \frac{1}{30}$ “
“ “ “ $\$105 = \frac{105}{30}$ “
$= 3\frac{1}{2}$ years.

Example 6. A certain sum at 6 per cent. per annum amounts to \$605 in $3\frac{1}{2}$ years: find the principal.

\$100 at 6 per cent. amounts, in $3\frac{1}{2}$ years, to \$121;
∴ \$121 is produced, in $3\frac{1}{2}$ years, by \$100,
\$1 “ “ “ $\$\frac{100}{121}$,
\$605 “ “ “ $\$\frac{100}{121} \times 605$
$= \$500.$

Therefore the required principal is \$500.

EXERCISE LX.

1. In what time will \$55.00 at 7 per cent. double itself?

2. Find the sum that will amount to £62 10s. at 5 per cent. per annum, simple interest, in 5 years.

3. At what rate will \$824.00 amount to \$997.04 in 3 years?

4. Find the amount of a note made on the 10th of Jan'y., 1882, for \$524.00 at 6 per cent. per annum, and payable on the 1st of September of the same year.

5. What sum must a father invest at 6 per cent. per annum, payable half yearly, to provide a half yearly income of \$750 for his daughter?

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161. \$100 will amount to \$107 in a year, if the interest be at 7 per cent.; hence \$100 is called the **Present Value** of \$107.

162. Discount is the difference between the given sum and the **Present Value,** in this case \$7.00, so that we have the following relations :

Present value + Discount of given sum = given sum,
Present value + int. of present value = given sum ;
$\therefore$ Discount of given sum = int. of p. value.

EXAMPLE 1. Find the present value of \$545 due in 6 months at 6 per cent.

Int. on \$100 at 6 per cent. for 6 months is \$3.00 ;

$\therefore$ Present value of \$103 = \$100
“ “ \$1 = $\$\frac{100}{103}$
“ “ \$545 = $\$\frac{100}{103} \times 545$
= $\$529.12\frac{64}{103}$

Note. Compare this example with example 6 (p. 172).

We can now get the discount by taking the present value from the given sum, $\$545 - \$529.12\frac{64}{103} = \$15.87\frac{39}{103}$, or we could get it independently as follows :

EXAMPLE 2. Find the discount on \$545 due 6 months hence at 6 per cent.

Interest on \$100 for 6 months at 6 per cent. is \$3 ;

$\therefore$ Discount of \$103 = \$3
“ “ \$1 = $\$\frac{3}{103}$
“ “ \$545 = $\$\frac{3}{103} \times 545$
= $\$15.87\frac{39}{103}$.

EXERCISE LXI.

1. Find the present value of \$821.50 due 8 months hence, interest at 7 per cent. per annum.

2. Find the discount on \$425 due 9 months hence, interest at 8 per cent. per annum.

3. A note is made for \$562.50 due in a year, interest at 8 per cent. per annum, what is the note worth at time of making it ?

4. A bill drawn on the 4th of Feb'y, 1881, for \$675.75 due in three months, is paid on the 20th of March of the same year, what is it worth, interest being at the rate of 7½ per cent. per annum?

◆

SECTION X.

163. The square of a given number is the product of the number by itself.

Thus 3 × 3 is the square of 3, or $9 = 3^2$. (67.)

The square root of a given number is a number which when multiplied by itself will produce the given number. Thus 3 is the square root of 9; for 3 × 3 = 9.

The square root of a number is frequently denoted by $\surd$ put before the number. Thus $\surd 9$ denotes the square root of 9.

Example 1. To find the square root of 625.

$$\begin{array}{r|l} & \dot{6}2\dot{5}(25 \\ & 4 \\ \hline 45 & 225 \\ & 225 \\ \hline \end{array}$$

Place a dot over the units' figure and over every second figure to its left and its right, thus dividing the number into periods of two figures each; remember that each period consists of the figure over which the dot is and the figure to its left. Find the greatest number whose square is contained in the first period on the left; this is 2, which place in the form of a quotient to the right of the number. Put 4, the square of 2, under the 6 and subtract it from the 6, and to the remainder 2 affix the next period 25, thus forming the number 225. Take twice 2, or 4, for a divisor; divide the 225, omitting the last figure by 4 and we obtain 5. Annex the 5 to the 2 before obtained, and to the divisor 4; then muliplying the 45 by the 5, we obtain 225, which being subtracted from the 225 before formed leaves no remainder; therefore 25 is the square root of 625.

Example 2. Find the square root of 4·2005.

$$\begin{array}{r|l} & \dot{4}\cdot2\dot{0}2\dot{5}(2\cdot05 \\ & 4 \\ \hline 405 & \quad 2025 \\ & \quad 2025 \\ \hline \end{array}$$

Place a point over the 4 the units' place, and over every second figure to the right. Thus in decimals we make the number of them even by adding a cipher if necessary.

After finding the first figure in the root, 4 will not divide 2 of the next period; we therefore put a cipher in the quotient and also in the divisor and affix to the 20 the next period 25 and proceed as before.

Example 3. To find the square root of $\frac{169}{289}$.

$$\sqrt{169} = 13, \ \sqrt{289} = 17;$$

therefore the square root of $\frac{169}{289} = \frac{13}{17}$.

Or $\frac{169}{289} = \cdot58477508\ldots$

$$\begin{array}{r|l} & \cdot5\dot{8}4\dot{7}7\dot{5}0\dot{8}\ldots\ (\cdot7647\ldots \\ & 49 \\ \hline 146 & \ \ 947 \\ & \ \ 876 \\ \hline 1524 & \qquad 7175 \\ & \qquad 6096 \\ \hline 15287 & \qquad 107908 \end{array}$$

Since there is no figure in the units' place, the first dot will be over the second place from the decimal point, &c.

Therefore the square root of $\frac{169}{289} = \cdot7647\ldots$

EXERCISE LXII.

Find the square root of:

1. 324
2. 94249
3. 379456
4. 531441
5. 158·76
6. 22·6576
7. 100·2001
8. 35·1649
9. ·128881
10. ·00822649
11. ·49112064

Extract the square root to 3 places of decimals of:

12. 5
13. ·3
14. ·03
15. 6·1
16. $\frac{5}{8}$

Find the value of:

17. $\sqrt{3} + \sqrt{5} + \sqrt{7}$
18. $\sqrt{8} + \sqrt{6}$
19. $\sqrt{20} + \sqrt{30} - \sqrt{50}$
20. $5\sqrt{\cdot02} + 4\sqrt{3\cdot8}$

SECTION XI.

RATIO.

164. If we wish to compare two numbers, as, for example, 30 and 6, we may say that their difference is 24. This is called Comparison by Difference.

But we may also compare them by finding how many times one contains the other, or what fraction one is of the other.

Thus, $30 \div 6 = \frac{30}{6} = \frac{5}{1}$, $48 \div 36 = \frac{48}{36} = 1\frac{1}{3}$, $15 \div 20 = \frac{15}{20} = \frac{3}{4}$. Hence we find that 30 is 5 times 6, 48 is $1\frac{1}{2}$ or $\frac{4}{3}$ of 36, 15 is $\frac{3}{4}$ of 20. This method is called Comparison by Ratio; and since a quotient can always be expressed by a fraction, we may say that the

165. Ratio of one number to another is the fraction which the former is of the latter.

The ratio of one number to another is often denoted by writing a semicolon between the numbers: thus, the ratio of 15 to 20 is written $15:20$; and, $15:20 = 3:4$, means that the ratio of 15 to 20 is equal to the ratio of 3 to 4. The ratio $12:12$, or $1:1$, is called a ratio of equality.

To compare concrete quantities by ratio we must express them by numbers having the same unit.

EXAMPLE 1. Find the ratio of 75 to 100.

$$75 \div 100 = \frac{75}{100} = \frac{3}{4}, \text{ or } 3:4.$$

EXAMPLE 2 What is the ratio of \$2.75 to \$4.50?

$$275 \div 450 = \frac{275}{450} = \frac{11}{18}, \text{ or } 11:18.$$

EXAMPLE 3. Find the ratio of $7\frac{1}{2}$ to 6.

$$7\tfrac{1}{2} \div 6 = \frac{15}{2} \times \frac{1}{6} = \frac{5}{4}, \text{ or } 5:4.$$

Example 4. Compare by ratio £1 5*s.* and £2 7*s.* 6*d.*

£1 5*s.* = 300*d.* ; £2 7*s.* 6*d.* = 570*d.* :

$300 \div 570 = \frac{300}{570} = \frac{10}{19}$, or 10 : 19.

Note 1. Quantities which are so different that they cannot be expressed as numbers having the same unit, or, in other words, cannot be reduced to the same denomination,c annot be compared by ratio. Thus, 1 foot and 1 lb. have no ratio to one another.

Note 2. A ratio may be expressed decimally : thus, $5 : 4 = \frac{5}{4} = 1{\cdot}25$, *i.e.* $5 : 4 = 1{\cdot}25 : 1$.

EXERCISE LXIII.

Find the following ratios in lowest terms :

1. 8 to 6.
2. 12 to 9.
3. 15 to 25.
4. 72 to 96.
5. 120 to 15.
6. 48 to 60.
7. $2\frac{1}{2} : 3\frac{1}{2}$.
8. $3\frac{3}{4} : 4\frac{1}{6}$.
9. $8\frac{1}{3} : 15$.
10. $7.50 : $10.
11. $3.75 : $5.25.
12. $15 : $9.60.

13. What is the ratio of £2 15*s.* to £3 6*s.* ?

14. Find the ratio of 8*s.* 9*d.* to 10*s.*

15. Find the ratio of 2 ac. 1 r. to 1 ac. 32 p.

16. Compare by ratio a pound Troy and a pound Avoirdupois ; also an ounce Troy and an ounce Avoirdupois.

17. Find the ratio of a sovereign ($$4.86\frac{2}{3}$) to $5.

18. A owns $\frac{3}{5}$ of a farm, B the remainder : what is the ratio of A's share to B's ?

19. Divide $60 between A and B, so that A's share may be to B's as 7 : 8.

MISCELLANEOUS PROBLEMS.

1. If 9 ac. 2 r. 27 p. cost \$1428, what will be the value of 26 ac. 3 r. 10 p. at the same rate?

$$9\text{ ac. }2\text{ r. }27\text{ p.} = 1547\text{ p.};\ 26\text{ ac. }3\text{ r. }10\text{ p.} = 4290\text{ p.}$$

$$\text{Value of }1547\text{ p.} = \$1428$$

$$\therefore\ \text{“}\quad\text{“}\quad 1\text{ p.} = \$\tfrac{1428}{1547}$$

$$\therefore\ \text{“}\quad\text{“}\quad 4290\text{ p.} = \frac{\$1428 \times 4290}{1547} = \$3960$$

2. If 3 ac. 2 r. cost \$350, what will be the cost of 5 ac. 3 r. 24 p.?

3. If 1 oz. 5 dwt. of silver be worth \$2.25, find the value of 1 lb. 5 oz. 11 dwt. 12 grs. of silver.

4. If the paving of 3 r. 6 p. of road costs \$1050, how much is that per mile?

5. If \$150 be paid for 2 ac. 1 r. 36 p. of land, how much could be bought at the same rate for \$5875?

6. If 1 dwt. of quinine be sold for 30c., how much is that per ounce Avordupois?

7. If a farm of 93 ac. 3 r. cost \$4500, how much should be paid for a farm containing 117 ac. 30 p., if 3 ac. of the former be worth 4 ac. of the latter?

$$93\text{ ac. }3\text{ r.} = 15000\text{ p.};\ 117\text{ ac. }30\text{ p.} = 18750\text{ p.}$$

$$1\text{ ac. of the former is worth }\tfrac{4}{3}\text{ ac. of the latter.}$$

$$\therefore\ 15000\text{ p. are worth }\tfrac{4}{3}\text{ of }15000\text{, or }20000\text{ p.}$$

$$\text{Now the value of }20000\text{ p.} = \$4500$$

$$\therefore\ \text{“}\quad\text{“}\quad 1\text{ p.} = \$\tfrac{45}{200} = \$\tfrac{9}{40}$$

$$\therefore\ \text{“}\quad\text{“}\quad 18750\text{ p.} = \$\tfrac{9}{40} \times 18750 = \$4218.75$$

8. If 1 oz. of gold coin be worth 15 oz. of silver coin, and the weight of a sovereign be 123 grs., what should be the weight of a shilling?

9. If 5 oxen are worth 3 horses, and 4 oxen cost $150, what will be the value of 5 horses?

10 If a farm containing 64 ac. 3 r. 35 p. be worth $2310, what should be paid for another farm of 230 ac. 1 r. 15 p., if 5 ac. of the former be worth 3 of the latter?

11. If 12 men can dig a drain 280 feet long in 4 days, how many men will be required to dig a similar drain 315 feet long in 3 days?

(1) 280 ft. are dug in 4 days by 12 men
(2) ∴ 1 ft. is “ 4 “ $\frac{12}{280}$ “
(3) ∴ 1 “ “ 1 day by $\frac{12 \times 4}{280}$ “
(4) ∴ 315 ft. are “ 1 “ $\frac{12 \times 4 \times 315}{280}$ “
(5) ∴ 315 “ “ 3 days by $\frac{12 \times 4 \times 315}{280 \times 3}$ “

$$\therefore \text{Required result} = \frac{12 \times 4 \times 315}{280 \times 3} = 18 \text{ days.}$$

Note. Lines (2) and (4) may be omitted when the learner is familiar with the Unitary method.

12. If 34 men can construct a road 1 furlong 15 p. long in 33 days, how many men will be required to make a road a mile long in 51 days?

13. If 17 men in 6 days earn $178.50, how much will 13 men earn in 15 days?

14. If 16 men dig a drain 300 yds. long in 13 days, in what time will 26 men dig a drain 525 yds. long?

15. If 3 pigs cost as much as 5 sheep, and 10 pigs and 12 sheep together cost $129, what should be paid for 8 pigs and 16 sheep?

1 pig is worth $\frac{5}{3}$ sheep, ∴ 10 pigs and 12 sheep = $\frac{86}{3}$ sheep ; and 8 pigs and 16 sheep = $\frac{88}{3}$ sheep.

Now the value of $\frac{86}{3}$ sheep $= \$129$
∴ " 1 " $= \$129 \times \frac{3}{86}$
∴ " $\frac{88}{3}$ " $= \$129 \times \frac{3}{86} \times \frac{88}{3} = \$132.$

16. If 2 oxen be worth 37 sheep, and 6 oxen and 17 sheep be worth $512, what will be the value of 9 oxen and 25 sheep?

17. If 3 bushels of wheat be worth as much as 5 bushels of barley, and 20 bushels of wheat with 20 bushels of barley sell for $30.40, how much money should be given with 24 bushels of wheat in exchange for 50 bushels of barley?

18. If 2 men can do as much work as 3 boys, how long long will it take 5 men and 6 boys to do what 6 men and 5 boys can do in 15 days?

19. Find the number of acres in an oblong field 374 feet long, 165 feet wide.

No. of sq. feet $= 374 \times 165$ (See 67.)

$$1 \text{ acre} = 4 \times 40 \times 30\tfrac{1}{4} \times 9 \text{ sq. feet}$$
$$= 4840 \times 9 \text{ sq. feet}$$
$$\therefore \text{No. of acres in field} = \frac{374 \times 165}{4840 \times 9} = \frac{34 \times 15}{40 \times 9}$$
$$= \tfrac{17}{12} = 1\tfrac{5}{12}.$$

20. Find the number of acres in an oblong 561 feet long, 264 feet broad.

21. A street 66 feet wide, is $2\frac{1}{2}$ miles long, find how many acres it contains.

22. A building lot 60 ft. 6 in. wide and 150 feet long is sold for $2000, how much is that per acre?

23. A person buys a farm of 100 acres at $48.40 per acre, and sells it in building lots 1 chain wide, 2 chains

long, at \$30 a lot. Supposing the streets to occupy $\frac{1}{4}$ of the land how much does he gain on the whole?

24. A board is 15 feet long, 11 inches wide: what is it worth at \$16.00 per thousand square feet?

25. A building lot is 176 feet wide: what must be its length, if its area is an acre?

26. A railway takes from a farm a strip $1\frac{1}{2}$ chains wide and $\frac{5}{8}$ mile long: what compensation must be paid at \$60 an acre?

27. Find the cost of papering the walls of a room 21 feet long, 20 ft. 4 in. wide and 12 ft. high, with paper at 45 cents a roll, 11 yards long and 15 in. wide, allowing for a door 7 ft. 9 in. by 4 ft., and two windows each 6 ft. by 4 ft.

Total length of wall $= 21 + 21 + 20\frac{1}{3} + 20\frac{1}{3}$
$= 82\frac{2}{3}$ feet; breadth $= 12$ feet;
$\therefore$ area of the walls $= 82\frac{2}{3} \times 12$ sq. ft $= 992$ sq. ft.
No. of sq. ft. to be deducted $= 7\frac{3}{4} \times 4 + 6 \times 4 \times 2$
$= 31 + 48 = 79.$
$\therefore$ No. of sq. ft. to be papered $= 992 - 79 = 913$
“ “ in roll $= 33 \times \frac{5}{4} = \frac{165}{4}$
$\therefore$ No of rolls required $= 913 \div \frac{165}{4} = \dfrac{913 \times 4}{165}$
$\therefore$ cost $= \dfrac{913 \times 4}{165} \times 45 = 996$ cents
$=$ \$9.96.

28. How many rolls of paper, 11 yards long and 15 in. wide will be required to paper the walls of a room 19 ft. long, 16 ft 6 in. wide and 10 ft. high, allowing for a door 7 ft. 3 in. by 4 ft., and two windows, each 6 ft. 9 in. by 4 ft.; and what will be the cost at 75 cents a roll?

29. How many yards of carpet, 2 ft. 3 in. wide, will cover a floor 24 ft. long and 18 ft. wide?

30. Find the cost of carpeting a room 20 ft. 3 in. long and 17 ft. 6 in. wide, with carpet 27 inches wide at $1 50 a yard?

31. Find the cost of painting the four walls of a room 22 ft. long, 16 feet wide and 11 ft. 3 in. high, at 85 cents a sq. yard.

32. A brick is 9 inches long, $4\frac{1}{2}$ inches wide and 3 inches thick: how many bricks will there be in a pile 45 feet long, 30 feet wide and 12 feet high?

No. of cubic feet in the pile $= 45 \times 30 \times 12$ (See 71.)

No of cubic inches in 1 brick $= 9 \times \frac{9}{2} \times 3$

∴ “ “ feet “ “ $= \dfrac{9 \times 9 \times 3}{1728 \times 2}$

∴ No. of bricks $= 45 \times 30 \times 12 \div \dfrac{9 \times 9 \times 3}{1728 \times 2}$

$$= \frac{45 \times 30 \times 12 \times 1728 \times 2}{9 \times 9 \times 3} = 230400$$

33. A pile of bricks is 40 feet long, 27 feet wide, 15 feet high: how many bricks does it contain?

34. How many bricks will be required to build a wall 135 feet long, 60 feet high and 18 inches thick?

35. How many cubic yards of earth will be removed in digging a drain $\frac{1}{4}$ mile long, 12 feet deep and 4 feet wide?

36. Find the cost of excavating a cellar 72 feet long, 28 feet wide and 10 feet deep at 75 cents a cubic yard.

37. Find the value of a piece of squared timber, 36 feet long, 1 ft. 9 in. wide, and 1 ft. 2 in. thick, at 24 cents a cubic foot.

38. If a flag-stone 10 ft. 6 in. long, 3 ft. 3 in. wide and 8 inches thick, cost $14, how much is that per cubic yard?

39. An imperial gallon contains 277·274 cubic inches: how many gallons will a cistern hold, its length being 5 feet, breadth 4 feet, and depth 3 feet 4 inches?

40. An imperial gallon of water weighs 10 lbs. Avoird. How much will the weight of a cubic foot of water differ from 1000 oz, ?

41. At what time between 4 and 5 o'clock will the hands of a clock be together ?

At 4 o'clock the h. hand is 20 minute spaces ahead.

Now the m. hand goes 60 m. spaces while the h. hand goes 5 spaces.

∴ the m. hand goes 12 spaces while the h. hand goes 1
∴ the m. hand gains 11 spaces in going 12
∴ “ “ 1 space “ $\frac{12}{11}$
∴ “ “ 20 spaces “ $\frac{240}{11}$
∴ the required time $= 21\frac{9}{11}$ m. past 4.

42. When will the hands of a clock be together between 2 and 3 o'clock ?

43. When first after 1 o'clock will the hands be directly opposite ?

44. When first after 3 o'clock will the hands be perpendicular to each other ?

45. At what times between 6 and 7 o'clock will the hands be perpendicular to each other ?

46. When will the hands be directly opposite between 9 and 10 o'clock ?

47. The hands are together between 7 and 8 : what is the exact time ?

48. A, B and C can walk round a course in 9, 11 and 15 minutes, respectively. They all start at the same time in the same direction, and continue walking till they are all together again. When will this happen and how many rounds will each have made ?

A goes $\frac{1}{9}$, B $\frac{1}{11}$ and C $\frac{1}{15}$ of a round in 1 min.
∴ A gains on B $\frac{1}{9} - \frac{1}{11}$, or $\frac{2}{99}$ round in 1 min.
∴ A gains on B 1 round in $\frac{99}{2}$ min.
A gains on C $\frac{1}{9} - \frac{1}{15}$, or $\frac{2}{45}$ in 1 min.
∴ A gains on C 1 round in $\frac{45}{2}$ min.

The L. C. M. of $\frac{99}{2}$ m. and $\frac{45}{2}$ m. = $\frac{495}{2}$ m. = 4 h. 7½ m.

$\frac{495}{2} \div 9 = \frac{55}{2} = 27\frac{1}{2}$ rounds for A.
$\frac{495}{2} \div 11 = \frac{45}{2} = 22\frac{1}{2}$ " " B.
$\frac{495}{2} \div 15 = \frac{33}{2} = 16\frac{1}{2}$ " " C.

49. A can run round a course in 6 min., B in 8 min. They start at the same time in the same direction. When will they first be together again, and how many rounds will each have made ?

50. If, in Ex. 49, A and B run in opposite directions, when and where will they first meet ?

51. A, B and C can walk round a course in 5, 7 and 11 min. respectively. If they all start together, in the same direction, when first will they be all together again ; and how many rounds will each have made ?

52. A can run 11 yards while B runs 9 yards. How many yards start can A afford to give B in a quarter of a mile race ?

53. A train 200 yards long took 45 seconds to cross a bridge 262 yards long At what rate was the train running ?

54. A can walk 15 yds. while B can walk 12 and C 9. If they all start together to walk round a course of 220 yards, how far will each have gone before they are all together again ?

55. Find the L C. M. of 135, 180, 72, 27 and 16.

56. Simplify $\dfrac{\cdot 2 \times \cdot 2 \times \cdot 2 - \cdot 01 \times \cdot 01 \times \cdot 01}{1 - \cdot 05 \times \cdot 05 \times \cdot 05}$.

57. A does $\frac{2}{9}$ of a work in 6 hours; B then assists and they finish the work in 12 hours. In what time could each do the work alone?

58. Multiply ·1875 by ·432 and verify the work by vulgar fractions.

59. Prove that $\cdot\dot{5}\dot{4} \times \cdot 5\dot{3} = \cdot\dot{2}\dot{9}0$.

60. Divide ·315 by 1·8; also 31·5 by ·28; and verify the work in each case by vulgar fractions.

61. When a certain regiment is formed into companies of 45, of 48, or of 60, there are always 12 men over. Find the number of men in the regiment.

62. Find the L. C. M. of $17\frac{1}{2}$, $9\frac{1}{3}$ and $6\frac{1}{4}$.

63. Simplify $(1\frac{3}{8} + 2\frac{5}{8}) \div (3\frac{7}{12} - 1\frac{5}{16})$.

64. If 5 be added to both terms of $\frac{7}{9}$, how much is its value altered?

65. How much must be subtracted from 4·6285 that when the remainder is divided by 87·6 the quotient may terminate in the fourth decimal place?

66. Which of the following quotients is more nearly correct $\frac{4\cdot162}{5\cdot7} = \cdot73$, or $\frac{4\cdot162}{\cdot73} = 5\cdot7$, and by how much?

67. Simplify $\frac{1}{2} + \frac{2}{3}$ of $\left\{1\frac{3}{4} - \frac{5}{6} \text{ of } (2 - \frac{1}{8})\cdot\right\}$

68. A owns $\frac{3}{5}$ of a farm, B $\frac{3}{4}$ of the remainder, and C the rest. A has 60 acres more than B and C together. How many acres has each?

69. The sum of the ages of A and B is 74 years, and $\frac{2}{3}$ of A's age is $\frac{7}{8}$ of B's: find their ages.

70. United States' gold is $\frac{9}{10}$ fine and English gold $\frac{11}{12}$ fine. If the value of the alloy be neglected, find the value of an English gold coin of the same weight as the U. S. gold dollar.

71. Prove that $\frac{5}{7} \times 3 = \frac{15}{7}$, and that $\frac{5}{7} \div 3 = \frac{5}{21}$.

72. If a pound of tea in one scale of a false balance weighs 1 lb. 2 oz., what would it weigh in the other scale ?

73. A person's net income after paying a tax of 15 mils on the dollar is $788 : what is the amount of his tax ?

74. By what vulgar fraction does $\cdot\dot{7}\dot{5}$ exceed ·75 ? Also find the difference between ·018182 and $\cdot0\dot{1}\dot{8}$.

75. Express £7 9*s.* 4½*d.* decimally in florins.

76. Express as a compound quantity 8·4375 of a pound Troy.

77. Find the cost of covering a floor, whose length is 6·75 metres and breadth 4.56 metres, with oilcloth at 4.65 francs per square metre.

78. Express decimally the ratio of a sovereign ($4.86⅔) to five dollars.

79. Express 1 m. 3 f. 27 p. decimally, with unit 1 chain.

80. The value of a certain decimal, when the unit is 1 lb. Troy, is 8 lbs. 9 oz. 15 dwt. 18 grs., what will be its value when the unit is £1 ?

81. What decimal of £3 7*s.* 6*d.* is £1 15*s.* ; and what is the unit of the result ?

82. Express 63.24 francs in English money, taking the franc = 9¾*d.*

83. A building lot is 1 chain wide and 2·85 ch. deep, and its area is said to be 1 r. 4 p. Find the error in the area.

84. A farm is 15 chains wide and contains 100 acres : shew that its perimeter is 2 miles 13 p. 2½ feet.

85. Compare $\frac{5}{9}$, $\frac{5\frac{1}{4}}{6}$ and $\frac{4}{5\frac{1}{3}}$.

86. Compare $\frac{3}{4}$, $\frac{3 \times 2}{4 \times 2}$, $\frac{3 + 2}{4 + 2}$ and $\frac{3 - 2}{4 - 2}$.

87. A can do half as much again as B, and one-third as much again as C, in the same time : in what time can all three together do a work which takes A alone 58 hours?

88. A, who owns $\frac{2}{3}$ of a ship, sells $\frac{3}{8}$ of his share for $4500. Find the value of the ship.

89. A man spends $\frac{3}{4}$ of his monthly wages in board and $\frac{2}{3}$ of the remainder in clothing and has then left $7.50. How much does he earn per month?

90. If the measure of a certain sum of money be $\frac{3}{5}$ when the unit is 1 franc, what will be its measure when the unit is £1, taking the franc = $9\frac{3}{4}$d.?

91. Find the quotient 3·17548 ÷ 92·475 to the nearest figure in the fifth place after the decimal point.

92. Three-fourths of A's money equals five-sixths of B's, and they together have $28.50 ; find how much each has.

93. For what sum should a cargo worth $2340 be insured, at $2\frac{1}{2}$ per cent., so that in case of loss the value of the cargo and the premium may both be recovered?

94. Find the value of £65 17s. 9d. in Canadian money, taking £1 = $$4.86\frac{2}{3}$.

95. Find the value of 78·3475 according as the unit is £1, an acre, or a pound Troy.

96. If $75 be the present worth of $80, due 9 months hence, what is the yearly rate of interest?

97. In what time will $120 amount to $135 at 9 per cent. per annum?

98. Express 5 h. 12 m. 45 sec. as a decimal of a day ; and 7 oz. 15 dwt. 4 grs. as a decimal of 1 lb. 12 oz. Avoird.

99. A man sold $\frac{3}{10}$ of his property, $\frac{4}{9}$ of the remainder, and $\frac{5}{8}$ of what was still left. What part of the property does he now own?

100. Express in Troy weight the sum of ·3125 of 1 lb. Troy and ·3875 of 1 lb. Avoird.

101. Multiply ·3456 by ·1234, and divide the result by 1·92.

102. What decimal of a pound Avoird. is 3·458 lb. Troy?

103. Shew that using ·66 for $\frac{2}{3}$ is less correct by $\frac{1}{600}$ than taking ·12 for $\frac{1}{8}$.

104. If the side of a square, to contain one acre, be found in yards to two places of decimals, shew that the area of the square so found will differ from an acre by less than 20 square inches.

105. A contractor undertakes to pave a road in 70 days and employs 48 men. After 28 days $\frac{1}{3}$ of the road is made, and the working day is reduced from 9 hours to 8 hours. How many more men must be put on in order to finish the work in time?

106. Divide 7·2 by ·1125, verifying the work by vulgar fractions.

107. A and B are shooting pigeons. A fires 5 shots to B's 3, but A kills only 1 in 3 shots, while B kills 1 in 2 shots: when B has killed 36 birds, how many will A have killed.

108. Find the interest on $875, lent March 7th, 1883, to be repaid July 31st, next, at 6 per cent. per annum.

109. If $\frac{4}{5}$ of a calf and $\frac{5}{8}$ of a sheep be each worth $\frac{1}{24}$ of an ox, and 12 calves, 10 sheep and 5 oxen are together worth $453, find the value of one of each.

110. Find by Practice the cost of 75 miles 3 f. 26 p. of railway, at $26400 a mile.

111. Reduce $\frac{3}{5} + \cfrac{2}{3 + \cfrac{1}{4 + \cfrac{1}{5\frac{2}{3}}}}$ to a simple fraction.

112. In how many days will the interest of \$73, lent at 8 per cent. per annum, be \$6 ?

113. Express 1 p. 4 yds. 1 ft. in metres, taking the metre = $39\frac{1}{3}$ inches.

114. If 3 men can do as much work in the same time as 5 boys ; and 5 men working 9 hours a day do $\frac{3}{10}$ of a work in 32 days : in how many days of 8 hours will 14 boys finish the work ?

115. If 1 oz. of gold be worth 15 oz. of silver and the weights of equal volumes of gold and silver be as 19 : 14, how many cubic inches of silver will be worth one cubic inch of gold ?

116. Find the number which has to $7\frac{1}{2}$ the same ratio as 10 has to 9.

117. How many boys must assist 2 men, so that a work which 4 men or 6 boys can do in 15 days, may be done in 6 days ?

118. Divide \$34.50 between A, B and C, so that A may have half as much again, and B one-third as much again as C.

119. A sovereign is given to pay a bill of 16·45 francs, find the change in French money, taking 1 franc = $9\frac{3}{4}d.$

120. Subtract from 7 lb. 5 oz. Troy weight, the sum of 1 lb. 8 oz. 17 dwt. 13 grs., 1 lb. 6 oz. 5 dr. 2 scruples and 1 10 oz. Avoird. weight, expressing the remainder in Troy weight.

121. A contractor undertakes to make a road one mile long in 90 days, and employs 72 men, but at the end of 60 days he finds he has made 6 furlongs. How many men may he discharge ?

122. What fraction is $3\cdot\dot{4}\dot{5}$ lbs. Av. of $4\cdot\dot{3}$ lbs. Troy ?

123. Divide 44 ac. 1 r. 28 p. 8 yds. by 23 ; also by 2 r. 14 p. $23\frac{1}{2}$ yds. Explain the results.

124. Simplify $1\frac{1}{2} + 2\frac{2}{3}$ of $3\frac{3}{4} \div 4\frac{4}{5}$; $(1\frac{1}{2} + 2\frac{2}{3}$ of $3\frac{3}{4}) \div 4\frac{4}{5}$ and $(1\frac{1}{2} + 2\frac{2}{3})$ of $3\frac{3}{4} \div 4\frac{4}{5}$.

125. Simplify $(1\cdot359 - \cdot175 \times \cdot0486) \div \cdot019$. State in words the operations indicated.

126. Divide ·56 by 8·75, 16·5 by ·352, and illustrate both operations by vulgar fractions.

127. A sets out from Weston for Toronto, a distance of 8 miles, at the same time that B starts from Toronto for Weston. A walks $3\frac{1}{2}$ miles an hour, B $2\frac{1}{2}$ miles. Where will they meet, and how far apart will they be 2 hours after starting ?

128. If $\frac{6}{7}$ and $\frac{2}{3}$ be expressed decimally to the nearest fig. in the third place, find how much the product of the two decimals will differ from $\frac{6}{7} \times \frac{2}{3}$.

129. Find the value of a farm containing 65·48 hectares, at 176·35 francs per hectare.

130. Convert 6·875 kilogrammes into Avoird. weight, taking 1 gramme $= 15\frac{1}{2}$ grs.

131. Find vulgar fractions, in lowest terms, equivalent to $\cdot\dot{1}8\dot{5}$, $\cdot3\dot{4}\dot{5}$ and $\cdot87\dot{6}$, respectively.

132. A sold to B ·2 of his stock, to C ·25 of the remainder, and to D ·3 of what was left. What part has he now ?

133. Compare $\frac{4\frac{1}{5}}{12}$, $\frac{\frac{15}{16}}{7\frac{1}{2}}$ and $\frac{7}{16\frac{1}{3}}$.

134. A, who owns $\frac{3}{8}$ of a property, sold $\frac{4}{9}$ of his share to B, who sold $\frac{4}{5}$ of his purchase to C for \$1200. Find the value of the whole property.

135. A, after he has done $\frac{3}{8}$ of a work alone in 6 hours, is assisted by B, and together they finish the work in 4 hours more. In what time could B alone do the whole work ?

136. Express 18s. $4\frac{1}{2}$d. as a decimal of a guinea.

137. A certain sum of money is divided among three persons, the first getting ·36 of the money, the second ·45 of the remainder, and the third \$6. How much do the first and second respectively receive ?

138. How many times can 3 lbs. 5 oz. 7 dwt 11 grs. be subtracted from 60 lbs., and how much will be left over ?

139. Find the H. C. F. and the L. C. M. of 765 and 884 ; and illustrate their use by simplifying $\frac{765}{884}$ and $\frac{1}{765} + \frac{1}{884}$.

140. State the meaning of $\frac{3}{8}$. What is the unit when $\frac{3}{8}$ths = £6 12s. $7\frac{1}{2}$d. ?

141. From the sum of $7\frac{3}{8}$ and $4\frac{5}{6}$ subtract the difference between $8\frac{4}{15}$ and $3\frac{7}{10}$, and divide the result by $7\frac{5}{18}$.

142. Divide the product of ·324 and ·216 by 17·28.

143. Define *measure* of a quantity. How can 180, 15 and $\frac{3}{4}$ be the measures of the same sum of money ?

144. A huckster buys 120 oranges at 3 for 5c. He sells half of them at 2 for 5c., and the rest at 4 for 5c., How much does he gain ?

145. A bath can be filled by the cold water tap in 20 min., and by the hot water tap in 30 min. The former is first opened, and 5 min. later the other is also turned on. In what time will the bath be filled ?

146. Reckoning interest at $7\frac{1}{2}$ per cent. per annum, find (1) the interest of \$584 for 83 days ; (2) the discount of \$590.40 due 4 months hence.

147. Find the vulgar fraction, in lowest terms, which is equivalent to $\cdot 61\dot{3}\dot{6}$. What vulgar fractions produce mixed recurring decimals ?

148. Find the value of 6·4375 when the value of the unit is £1 12*s.*

149. A man spent $135 more than a third of his money in buying a site, and $325 more than two-fifths in building a house, which left him one-sixth for furnishing. How much money had he?

150. A's money is $\frac{5}{6}$ of B's, but if A had two dollars more he would have $\frac{7}{8}$ of B's. How much has each?

151. A sum of money is divided among A, B and C, so that A has ·38 of it, B ·27 of the remainder and C $22.63. Find the shares of A and B.

152. A certain length, measured by a yard rule $\frac{1}{4}$ inch too short, appears to be 27 yards: what would it have measured, if the rule had been $\frac{1}{4}$ inch too long?

153. The measure of a certain weight is 9·648 when the unit is 1 oz. Avoirdupois; what would be its measure if 1 oz. Troy were the unit?

154. A cricket ground is 528 feet long and 495 feet broad. How much would it cost to fence it round at $2.40 a rod, and to sod it at 7½c a square yard.

155. A walks 20 yards while B walks 24 and C 18. They all start together to walk round a course which C can go over in 5 minutes. If they continue walking, when will they be all together again?

156. A boy devotes one-third of his study time to arithmetic, three-eighths of the remainder to grammar, and then has 35 minutes left for the rest of his lessons. How much time does he give to study?

157. Taking the weight of a cubic foot of water to be 1000 oz. and ice to be one-tenth lighter than the same bulk of water, find the number of tons in an acre of ice 9 inches thick.

158. If £1, 25·55 francs and 4.86\frac{2}{3}$ are all equivalent, express the value of a franc in cents, and that of a dollar in francs.

ANSWERS.

EXERCISE I. (PAGE 3).

1. Fifty-six, Seventy-eight, Eighty-three, Ninety-five, Seventy, forty-nine, Twenty-six, Thirty-four, Sixty-seven.

2. 29, 18, 36, 48, 54, 65, 76, 89, 92.

3. One hundred and eight, One hundred and ten, One hundred and fifteen, One hundred and eighteen, One hundred and twenty-four, One hundred and thirty-six, Two hundred and five, Two hundred and ten, Two hundred and Twelve, three hundred and sixteen, Three hundred and forty, Four hundred and sixty-five, Five hundred and eighty-seven, Six hundred and nine, Seven hundred and twenty-eight, Eight hundred and eighty-eight, Nine hundred and sixty-four.

4. 106, 210, 340, 465, 504, 678, 742, 816, 987.

EXERCISE II. (PAGE 4).

1. One thousand two hundred and six, One thousand three hundred and fifty-seven, Two thousand six hundred and fifteen, Three thousand and eight, Four thousand six hundred, Five thousand seven hundred and eighty, Six thousand, Seven thousand eight hundred and twelve, Eight thousand five hundred and seventy-nine, Nine thousand nine hundred and ninety-nine.

2. 1400, 2650, 3048, 4000, 5709, 6254, 7000, 8090, 9009.

3. Sixteen thousand four hundred, Thirty-five thousand and eight, Seventy thousand four hundred and sixty-three,

Eighty-five thousand, Ninety-four thousand six hundred, One hundred and five thousand six hundred and thirty-four, Three hundred and sixty-eight thousand, Four hundred thousand nine hundred and eighty, Five hundred thousand six hundred, Six hundred and forty-three thousand five hundred and seventy-eight, Seven hundred and fifty thousand eight hundred and sixty, Eight hundred thousand and eight, Nine hundred and nine thousand and ninety-nine.

4. Three thousand five hundred and sixty-eight, Four thousand and four, Seven thousand and eighty-nine, Eighteen thousand six hundred, Forty-eight thousand and sixty-five, Eighty-two thousand and twenty, Ninety-five thousand and ninety-five, One hundred and sixty thousand eight hundred and forty, Two hundred and seven thousand five hundred and thirty-six, Three hundred and fifteen thousand, Six hundred thousand and twenty-eighty, Seven hundred and one thousand one hundred and eleven, Eight hundred and fifty-four thousand nine hundred, Nine hundred and ninety-nine thousand nine hundred and ninety-nine, Nine hundred and nine thousand and ninety.

5. 71439, 405607, 100100, 880880, 144000, 900990, 80008, 560070.

EXERCISE III. (PAGE 7).

1. Seven million, Twenty-five million sixty-four thousand seven hundred and eighty, Sixty five million four hundred and eighty thousand seven hundred, Eight hundred and twelve million four thousand and five, Seven billion five hundred million six hundred and fifty thousand.

2. 5006007, 14085016, 900800700, 3020001090, 12000570203.

3. Four million and five, two hundred and five million eight hundred, Sixteen billion six hundred thousand and seventy-two, Eight hundred and seventy billion nine million eighty thousand and sixty, Three hundred and twelve billion five hundred and seven million six hundred and eighteen thousand.

4. 25000025, 4016000015.

5. 0, 10, 100, 1000, 10000, 100000, 1000000.

6. 9, 99, 999, 9999, 99999, 999999, 9999999.

7. One million one hundred and eleven thousand one hundred and eleven, One hundred and one million ten thousand one hundred and one, Two billion two million two thousand and two, Three hundred million thirty thousand and three, Forty billion four hundred thousand and four, One billion two hundred and thirty-four million five hundred and sixty-seven thousand eight hundred and ninety, Five trillion six million and six.

EXERCISE IV. (PAGE 8).

1. 16, 25, 33, 49, 99, 160, 232, 400, 600.

2. 650, 1660, 2720, 1387.

3. 1882, 1560, 1650, 2666, 1449.

EXERCISE V. (PAGE 9).

1. XLVIII., XCVI., CLXII., CCCXLV., DXLVII., DCCCIX., DCCCC., DCD., DCDXCIX., DCCCC.

2. MVIII., MCCXXXIV., MCCCLVII., MMCDLXVIII., MMMLX., MXII., MD.

3. $\overline{\text{IV}}$., $\overline{\text{V}}$CCCLXVIII., $\overline{\text{XI}}$XLV., $\overline{\text{XXVII}}$DCCC., $\overline{\text{DCXXX}}$.

EXERCISE VI. (Page 12).

1. 20. 2. 20. 3. 33. 4. 15 marks. 5. 20c. 6. 21.

EXERCISE VII. (Page 13).

1. 25. 2. 27. 3. 24. 4. 24. 5. 54. 6. 63.
7. 52. 8. 44. 9. 84. 10. 98. 11. 170. 12. 329.
13. 643. 14. 510. 15. 2626. 16. 1204. 17. 4243.
18. 18288. 19. 11457. 20. 24187. 21. 14741.
22. 12126. 23. 4257. 24. 9880. 25. 31 ; 63 ; 240.
26. 136 persons. 27. 170 acres. 28. 172 persons.
29. 73402. 30. 4662. 31. 118 runs. 32. 35 persons.
33. 69 eggs. 34. 74 cents. 35. 325.

EXERCISE VIII. (Page 17).

1. 7130. 2. 11808. 3. 4444. 4 20183. 5. 21072.
6. 25348. 7. 423. 8. 17262. 9. 5328. 10. 2470.
11. 15222. 12. 51908. 13. 110069. 14. 1725.
15. 499 men. 16. 435 feet. 17. $813. 18. $322.
19. 147 pupils. 20. $684. 21. $210. 22. 144.
23. $27750. 24. $12139436.

EXERCISE IX. (Page 21).

1. 7 cents. 2. 7 pupils. 3. 9 feet. 4. 6 cents. 5. 7
words. 6. 14 cents. 7. 5, 5, 11, 7, 5.

EXERCISE X. (Page 22).

1. 12. 2. 10. 3. 11. 4. 22 5. 24. 6. 41. 7. 7.
8. 9. 9. 89. 10. 82. 11. 286. 12. 155. 13. 89.
14. 92. 15. 10. 16. 81. 17. 75. 18. 117. 19. 1.
20. 236. 21. 21. 22. 20. 23. 93. 24. 87. 25. 78.
26. 20. 27. 59. 28. 44. 29. 89. 30. 39. 31. 89.
32. 47. 33. 24, 26, 13, 25. 34. 32, 43. 35. 78 sheep ;
10 sheep. 36. 36 beech-nuts. 37. 53, 9. 38. 14 cents.

EXERCISE XI. (PAGE 24.)

1. 1112. 2. 1142. 3. 1218. 4. 29393. 5. 7888. 6. 34838. 7. 87468. 8. 56105. 9. 187867. 10. 8929. 11. 1118. 12. 2718. 13. 201995. 14. 72699. 15. 370917. 16. 78467. 17. 78273. 18. 99253. 19. 8897778. 20. 778878. 21. 489869. 22. 778681. 23. 111689. 24. 1000. 25. 1798; 305008. 26. 161329. 27. 8662. 28. 1904; 75 years. 29. 18514. 30. 25 years. 31. 15362 bushels. 32. 9988 men. 33. 382 scholars. 34. 1815. 35. $58 saved. 36. 29002. 37. 652. 38. $1421711. 39. 985002. 40. 385012. 41. 84865. 42. 459788.

EXERCISE XII. (PAGE 28).

1. 35 cents. 2. 30 pages. 3. 28 miles. 4. 108 months. 5. 3 cents. 6. 3 cents. 7. 80, 55, 54, 43, 2, 59, 28.

EXERCISE XIII. (PAGE 29).

1. 106. 2. 94. 3. 176. 4. 144. 5. 180. 6. 87. 7. 108. 8. 332. 9. 450. 10. 335. 11. 216. 12. 594. 13. 189. 14. 371. 15. 360. 16. 616. 17. 621. 18. 486. 19. 200. 20. 990. 21. 583. 22. 957. 23. 720. 24. 588. 25. 1374. 26. 2400. 27. 1104. 28. 3885. 29. 2982. 30. 6335. 31. 6680. 32. 4383. 33. 5600. 34. 5918. 35. 8448. 36. 225 bushels, 29250 cents. 37. 825 steps. 38. 288 gallons. 39. 4125 pages.

EXERCISE XIV. (PAGE 30).

1. 11698. 2. 29619. 3. 24228. 4. 485340. 5. 416160. 6. 404825. 7. 3073630. 8. 388064. 9. 346284. 10. 590592. 11. 833184. 12. 234927. 13. 1098444. 14. 130061614; 325154035; 195092421;

455215649 ; 260123228 ; 585277263 ; 390184842 ;
520246456 ; 715338877 ; 780369684. 15. 450 miles.
16. 40 miles. 17. 170 miles. 18. 315 miles. 19. $91.
20 804 miles. 21. 1980000.

EXERCISE XV. (PAGE 33).

1. 23265. 2. 221040. 3. 38280. 4. 741600.
5. 4830. 6. 15312. 7. 1011600. 8. 57960.
9. 1168000. 10. 91692. 11. 199500. 12. 720000.
13. 6120000. 14. 2632. 15. 23625. 16. 180792.
17. 2842. 18. 4416. 19. 27945. 20. 9728. 21. 720.
22. 64. 23. 243. 24. 256. 25. 125.

EXERCISE XVI. (PAGE 34).

1. 3575. 2. 8786. 3. 20532. 4. 20894. 5. 26566.
6. 41817. 7. 64090. 8. 54473. 9. 279736. 10. 536544.
11. 6374905. 12. 36825843. 13. 41574225.
14. 34683600. 15. 46713480. 16. 29578864.
17. 15627520. 18. 29049216. 19. 20641056.
20. 40064304. 21. 97458000. 22. 99001250.
23. 9179693320. 24. 12662836704. 25. 24708428198.
26. 1727943030. 27. 91477415850. 28. 153041719600.
29. 110899152000. 30. 8072398400. 31. 141597468015.
32. (1) 14882220, 330716, 413395, 744111 ; (2) 90160884,
678037, 834823, 910716. 33. 167349 dollars.
34. 11708 dollars. 35. 6570000 grains. 36. 938000 cents.
37. 46332 cents. 38. 1584 rails. 39. 12000 feet.
40. 2092500 dollars.

EXERCISE XVII. (PAGE 37).

1. 281488. 2. 1225713. 3. 742438. 4. 3772820.
5. 34433432. 6. 26941376. 7. 4514544. 8. 5869068.
9. 16594515. 10. 23206596. 11. 1471340368.

12. 47090826324. 13. 14758631995. 14. 30097343824.
15. 49268476142. 16. 56110851600. 17. 467250000.
18. 4446702000. 19. 18914656. 20. 1263256140.
21. 222094000. 22. 2106837550. 23. 40904843200.
24. 215506692. 25. 33758471264068.
26. 5759935172223. 27. 5662809628416.
28. 2910195338300. 29. 6341789969900.
30. 4897462869387. 31. 3009957855464
32. 3030636904885. 33. 4696902344957.
34. 7902873599770. 35. 8492452 and 56651448.
36. 111320 square yards. 37. 71000 matches.
38. 649250 letters. 39. 4032000 feet. 40. 2412117318.
41. 15240825. 42. 231679839.

EXERCISE XVIII. (PAGE 42).

1. 8 boys. 2. 9 ranks. 3. 8 lbs., 4 cents. 4. 6 weeks.
5. 12 times. 6. 8 days. 7. 11 pages. 8. 7 cents.
9. 6 cents. 10. 7 times.

EXERCISE XIX. (PAGE 43).

1. 3421. 2. 3827. 3. 2399. 4. 1232. 5. 1523.
6. 3759. 7. 2102. 8. 2304. 9. 2884. 10. 1020.
11. 1506. 12. 1729. 13. 1164. 14. 1351. 15. 1022.
16. 1194. 17. 129. 18. 146. 19. 1020. 20. 822.
21. 867. 22. 376 – 8. 23. 742. 24. 534 – 2. 25. 132.
26. 871. 27. 315 – 9. 28. 75, 5 cent pieces.
29. 2640 steps. 30. 64 days. 31. 416 dollars. 32. 5491.
33. 127 times. 34. 105 cents. 35. 52 pounds. 36. 165
feet. 37. 480 seconds.

EXERCISE XX. (PAGE 46).

1. 158. 2. 176. 3. 338. 4. 1357. 5. 625. 6. 769.
7. 1922. 8. 2406. 9. 3687. 10. 3812. 11. 7623.

12. 6125 – 6. 13. 779 – 2. 14. 2439 – 5. 15. 2777 – 7.
16. 5293. 17. 3677 – 2. 18. 496 – 8. 19. 90909.
20. 24691 – 1. 21. 88888 – 8. 22. 77777 – 7. 23. 5716.
24 8. 25. 3 times. 26. 46099. 27. 4 times. 28. 4.
29. 16 pounds. 30. 38 yards, and 7 cents over.

EXERCISE XXI. (Page 49).

1. 395 – 10. 2. 949 – 16. 3. 3049. 4. 2144.
5. 1275. 6. 1191. 7. 7534 – 36. 8. 2377. 9. 2435.
10. 874. 11 11326 – 26. 12. 1790 – 26. 13. 3481 – 103.
14 994 – 52. 15. 3948 – 119. 16. 2842 – 76.
17. 66 – 54. 18. 1054 – 517. 19. 358 – 133.
20. 16 – 35875. 21. 3 × 3 × 5 × 5. 22. 5 × 5 × 5 × 5.
23. 3 × 5 × 7. 24. 5 × 11 × 13.

EXERCISE XXII. (Page 51).

1. 118. 2. 123. 3. 124 4. 223. 5. 132.
6. 1307. 7. 213. 8. 234. 9. 308. 10. 419.
11. 324. 12. 109. 13. 351. 14. 405. 15. 572.
16. 205. 17. 36. 18. 53. 19. 198 – 160. 20. 657 – 65.
21. 104 times. 22. 264. 23. 508. 24. 7544. 25. 257.
26. 1992393. 27. 972. 28. 455 bushels. 29. 19 hours.
30. 28 bushels. 31. 4 dollars.

EXERCISE XXIII. (Paoe 54).

1. 386. 2 476. 3. 8667. 4. 845. 5. 421 – 5.
6. 9009. 7. 9374 – 16. 8. 13662. 9. 5096.
10. 2882 – 436. 11. 13208. 12. 917 – 1084.
13. 3853 – 801. 14 223 – 3400. 15. 340 – 867.
16. 71 – 1940. 17. 573 – 3139. 18. 659 – 332.
19. 7315. 20. 1063 – 2767. 21. 66 – 784. 22 60.
23 87 times. 24 146. 25. 1094. 26. 532 bushels.

27. 173. 28. 120 minutes. 29. 13. 30. 58 boys
31. 198 bush. – 1. 32. 7 days. 33. 52800 rails.
34. 9982, 9380. 35. 16 dollars. 36. 5876. 37. 15, 36.
38. 240 miles W. of Montreal. 39. 58 papers. 40. 48 miles a day 41. 6 hours after meeting the freight train.
42. Carriage, 322 dollars ; horse, 206 dollars ; harness, 160 dollars. 43. 19 seconds.

EXERCISE XXIV. (PAGE 57).

1. 3. 2. 2304. 3. 37. 4. 432. 5. 61, 83.
6. 13 nuts and 8 over. 7. 13776. 8. 24.

MISCELLANEOUS EXAMPLES. (PAGE 58.)

PAPER I.

2. 414781192. 3. 419067188440 and 2683298199 – 96.
4. 1890981, One million eight hundred and ninety thousand nine hundred and eighty-one. 5. 893395000 and 250 – 1. 6. 624 dollars.

PAPER II.

2. 10 hours. 3. 12090972, Twelve million ninety thousand nine hundred and seventy-two. 4. 374 – 223.
5. One thousand eight hundred and eighty-two, 1882, MDCCCLXXXII. 6. 671484 and 67148 – 4.

PAPER III.

2. 984 and 1179. 3. 2176904 hours – 32. 4. 2378.
5. Forty-nine thousand nine hundred and fifty, and 49950.
6. 2038.

PAPER IV.

2. 537915, 536389, 409846976,, 704 3. 22265, 61.
4. 57 years 5. 7051968, 6654071. 6. 9090909 bricks.

PAPER V.

1. 8757 sovereigns. 2. 7642 trees. 3. 71478. 4. 78 strokes. 5. 313 passengers. 6. 4811582 passengers.

PAPER VI.

1. 17324026. 2. 59655. 4. 3686596. 5. 4352080. 6. 665484.

EXERCISE XXV. (PAGE 66).

1. $115.82. 2. $103.20. 3. $3.75. 4. $6.34. 5. $15.75. 6. $64 86. 7. $4 25. 8. $7 02 and 19 cents – 18. 9. $3.63. 10. 252 bushels. 11. $3.52. 12. $105.00. 13. $6.15. 14. 24 bushels 15 $4375. 16. 43 pounds. 17. All but $1.65. 18. $6.24. 19. A gives B 16, 25-cent pieces and B gives A 4, 20-cent pieces.

EXERCISE XXVI. (PAGE 69).

1. 5 oranges. 2. 16 cents. 3. 4 hours. 4. 20 minutes. 5. 20 days. 6. $1.44 7. 5 lbs 8. $45. 9 20 cents. 10. 20 yds. 11. 20 yds. 12. 40 yds. 13. 48 yrs., 12 yrs. 14. 45 cents. 15. $5.44. 16. $210. 17. 64925 feet. 18. 60 yards. 19. 5 seconds. 20. A 24 years ; B 12 years ; C 6 years. 21. 40704. 22. $75 and $45 respectively. 23. $36 24. In 4 minutes more. 25. $481, $419

EXERCISE XXVII. (PAGE 83).

1. 157026 feet. 2. 608487 oz. 3. $776909\frac{1}{4}$ feet. 4. 18345 mins. 5. £253 10*s*. 10*d*. 6. 345555000 sec. 7. 35 cwt. 1 qr. 18 lbs. 9 oz. 11 drs. 8. 8708 pts. 9. 15151 bushels. 10. 4810752 c. in. 11. 29411 bushels, 26 lbs. 12. 539832 grs. 13. 1 a. 2 r. 15 per. 10 yds. 0 ft. 30 in. 14. 63723 grs. 15. 2014 bushels 38 lbs. 16. 44120 grs. 17. 7 fur. 6 per. 5 yds. 0 ft. 4 in. 18. 3 fur. 34 per. 2 yds. 2 ft. 19. 31556929 seconds.

EXERCISE XXVIII. (PAGE 85).

1. $1811.92. 2. 57 lbs. 10 ozs. 2 dwt. 12 grs.
3. 73 mls. 3 fur. 11 per. 2 ft. 4. £279 10*s.* $7\frac{1}{4}$*d.*
5. 88 lbs. 7 ozs. 3 drs. 2 scr. 15 grs. 6. $3747.73.
7. 688 tons 11 cwt. 2 qrs. 14 lbs. 13 ozs. 15 drs.
8. 969 yds. 2 qrs. 1 nl. 9. 26 wks. 6 dys. 7 hrs. 45 mins.
10. 1765 c. yds. 12 ft. 1083 inches. 11. 1837 a. 2 r.
13 per. 0 yds. 5 ft. 55 inches. 12. $2987.60.
13. 85 lbs. 5 ozs. 3 drs. 2 scr. 13 grs. 14. 162 bushels
2 pks. 1 gal. 15. $10546.07.

EXERCISE XXIX. (PAGE 87).

1. $84.89. 2. £13 17*s.* $10\frac{1}{4}$*d.* 3. 16 tons, 5 cwt. 2 qrs.
21 lbs. 9 ozs. 4. 7 yrs. 46 wks. 4 dys 9 hrs. 10 mins.
5. 4 lbs. 10 ozs. 4 drs. 0 scr. 17 grs. 6. 27 bu. 2 pks.
1 gal. 7. 1 lb. 10 ozs. 10 dwts. 21 grs. 8. 9 mls. 6 fur.
33 per. 2 yds. 0 ft. 9 in. 9. 2 c. yds. 20 c. ft. 1139 c. in.
10. 8 yds. 2 qrs. 2 nls. 11. 9 a. 1 r. 20 per. 5 yds. 0 ft.
77 in. 12. 7 cwts. 85 lbs. 9 ozs. 10 drs. 13. $188.68.

EXERCISE XXX. (PAGE 89).

1. £186 7*s.* 1*d.* 2. 76 lbs. 3. $3189.75 4. 81 cwts.
2 qrs. 20 lbs. 8 ozs. 5. 71 mls. 4 fur. 29 per. 1 ft. 6 in.
6. 100 lbs. 2 oz. 1 scr. 10 grs. 7. 147 dys. 15 hrs. 22 mins.
8. 1102 a. 2 r. 33 per. 8 yds. 9. 700 yds. 3 qrs. 1 nl. 2 in.
10. 12032 bushels, 2 pks. 1 gal. 3 qts. 11. 81449 dys.
0 hrs. 55 mins. 51 secs. 12. 659742 c. yds. 6 ft. 672 in.
13. 51622 yrs. 0 wks. 2 dys. 16 hrs. 14. 184 per. 5 yds.
15. 47 yds. 3 qrs. 16. 14738 lbs. 15 ozs. 17. $10.47.
18. $722.50 19. 149 yds. 0 qrs. 1 nl. 20. 15 cwt. 3 qrs.
18 lbs. 4 ozs. $1\frac{149}{875}$ drs.

EXERCISE XXXI. (PAGE 92).

1. £149 4*s.* $2\frac{1}{4}$*d.* and $\frac{4}{11}$ over. 2. 1 a. 0 r. 10 per.
4 yds. 2 ft. 3. 1 cwt. 0 qrs. 16 lbs. 15 oz. $6\frac{6}{7}$ drs.
4. 1 oz. 4 dwt. $20\frac{6}{55}$ grs. 5. 1 ft. $1485\frac{27}{121}$ in.
6. $2.08. 7. 9 times. 8. £98 2*s.* $4\frac{1}{2}$*d.*
9. 3738 sovereigns. 10. $94\frac{7}{8}$ yds. 11. 4 c. yds.
20 ft. 635 2 in. 12. 55 yds.

MISCELLANEOUS EXAMPLES.

PAPER I. (PAGE 93).

1. $146\frac{1}{4}$ lbs. 2. 923 tons 16 cwt. 0 qrs. 8 lbs. 3. 43119 in.
4. £714450 7*s.* $4\frac{1}{4}$*d.* 5. 15782 mls. 6 fur. 24 per. 5 yds.
2 ft. 4 in. 6. 4 lbs. 1 oz. 9 dwt. $22\frac{106}{125}$ grs.

PAPER II. (PAGE 94).

1. 3430 rods. 2. 7.37\frac{7}{12}$. 3. $38\frac{1}{8}$ cts. 4. 34 chests.
5. 18 yds. 6. 250000 sq. links.

PAPER III. (PAGE 94).

1. 182 days. 2. £95. 3. 3738 sovereigns.
4. 91132497, 506,909. 6. 50 acres, $175.

PAPER IV. (PAGE 95).

1. $1267\frac{1}{5}$ times. 2. 90000 lbs. Avoir. 3 $3.60.
4. 4c yds. 20 ft. 635·2 in. 5. 62 pupils, $83\frac{1}{3}$ c. ft.
6. $853.50.

PAPER V. (PAGE 95).

1 1 oz. 11 dwt. $1\frac{7}{17}$ grs. 2. 1 a. 10 per.
4 yds. 2 ft. 3. 20 miles per hour. 4. $72.50.
5. $1341.60. 6. $723.14.

EXERCISE XXXII. (PAGE 97).

1. 32. 2. 999 3. 413. 4. 901. 5. 608. 6. 839.
7. 19. 8. 1. 9. 101. 10. 292. 11. 10111. 12. 1.
13 44. 14. 1103. 15. 909. 16. 3.
17. A, 34 ; B, 46 ; C, 29.

EXERCISE XXXIII. (PAGE 100).

1. 105. 2. 112. 3. 300. 4. 120.
5. 5016. 6. 9196. 7. 1140. 8. 504.
9. 3465 10. 96. 11. 1482. 12. 1260.
13. 180. 14. 240. 15. 16575. 16. 27000.
17. 120. 18. 45045. 19. $10.50. 20. 720 soldiers.
21. 365. 22. 366 feet. 23. 9 doz.

MISCELLANEOUS EXAMPLES.

PAPER I. (PAGE 101).

1. In 5 seconds. 2. 125, 498, 327. 3. 1155, 1716.
4. 609. 5. 4 times in one sec. 6. 352 times.

PAPER II. (PAGE 101).

1. 105. 2. $\$1.74\frac{4}{5}$, $17.48. 3. 704.
4. No. 5. When G.C.F. is 1. 6. 27720.

PAPER III. (PAGE 102).

1. 22658190935. 2. 3507. 3. 1287, 9009.
4 508928035. 6. $478\frac{1}{2}$.

PAPER IV. (PAGE 102).

1. 51 bushels, 54 lbs. 2. 1200 yrs. 3. 297 bushels, 9 lbs.
4. $240.67. 5. 120777. 6. $\$1.33\frac{1}{3}$.

EXERCISE XXXIV. (PAGE 107).

1. $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}$. 2. $\frac{5}{8}, \frac{3}{8}$. 3. $\frac{7}{4}, 1\frac{3}{4}$. 4. $\frac{2}{5}$. 5. $\frac{3}{8}$.
6. $\frac{5}{6}$ is greater by $\frac{1}{12}$. 7. $\frac{2}{5}$ of $\frac{1}{4}$. 8. $\frac{8}{5}, \frac{12}{5}, \frac{23}{8}, \frac{33}{8}$,
$\frac{43}{8}, \frac{125}{9}, \frac{106}{17}$, 9. $1\frac{1}{3}, 1\frac{1}{5}, 1\frac{3}{8}, 3\frac{4}{7}, 2\frac{5}{12}, 2\frac{3}{16}, 3\frac{4}{15}, 5\frac{5}{19}$.
10. 1, 2, 3, 2, 4, 8, 6, 7. 11. $\frac{4}{4}, \frac{12}{4}, \frac{20}{4}, \frac{32}{4}, \frac{52}{4}, \frac{80}{4}$.
12. $\frac{65}{5}, \frac{70}{5}, \frac{75}{5}, \frac{80}{5}, \frac{100}{5}, \frac{125}{5}$. 13. $\frac{5}{16}, \frac{24}{16}, \frac{44}{16}$. 14 3 lbs.

EXERCISE XXXV. (PAGE 109).

1. $\frac{3}{4}$. 2. $\frac{3}{5}$. 3. $\frac{3}{5}$. 4. $\frac{2}{3}$. 5. $\frac{3}{7}$. 6. $\frac{3}{5}$.
7. $\frac{3}{5}$. 8. $\frac{7}{20}$. 9. $\frac{9}{17}$. 10. $\frac{5}{8}$. 11. $\frac{7}{8}$. 12. $\frac{3}{5}$.
13. $\frac{4}{5}$. 14. $\frac{103}{136}$. 15. $\frac{31}{84}$. 16. $\frac{945}{1529}$. 17. $\frac{1}{5}, \frac{7}{20}, \frac{3}{4}, \frac{4}{5}$.
18. $\frac{1}{10}$. 19. $\frac{5}{8}$. 20. $\frac{2}{3}$. 21. $\frac{1}{4}$. 22. $\frac{3}{4}$. 23. $\frac{1}{5}, \frac{2}{5}, \frac{4}{5}$.

EXERCISE XXXVI. (PAGE 111).

1. $\frac{3}{12}, \frac{9}{12}$. 2. $\frac{20}{24}, \frac{9}{24}$. 3. $\frac{5}{15}, \frac{3}{15}$. 4. $\frac{15}{36}, \frac{16}{36}$.
5. $\frac{12}{20}, \frac{25}{20}$. 6. $\frac{11}{22}, \frac{10}{22}$. 7. $\frac{6}{12}, \frac{4}{12}, \frac{3}{12}$ 8. $\frac{12}{30}, \frac{9}{30}, \frac{14}{30}$.
9. $\frac{35}{105}, \frac{21}{105}, \frac{15}{105}$. 10. $\frac{70}{105}, \frac{84}{105}, \frac{90}{105}$. 11. $\frac{18}{24}, \frac{15}{24}, \frac{4}{24}, \frac{14}{24}$.
12. $\frac{4}{30}, \frac{21}{30}, \frac{25}{30}, \frac{10}{30}$. 13. $\frac{1040}{1440}, \frac{1020}{1440}, \frac{528}{1040}, \frac{945}{1440}$.
14. $\frac{15}{84}, \frac{18}{84}, \frac{32}{84}, \frac{35}{84}$. 15. $\frac{24}{36}, \frac{16}{36}, \frac{3}{36}, \frac{14}{36}$.
16. $\frac{280}{420}, \frac{105}{420}, \frac{252}{420}, \frac{240}{420}$. 17. $\frac{8}{16}, \frac{12}{16}, \frac{14}{16}, \frac{9}{16}$.
18. $\frac{375}{625}, \frac{100}{625}, \frac{35}{625}, \frac{11}{625}$. 19. $\frac{27}{81}, \frac{9}{81}, \frac{3}{81}, \frac{1}{81}$.
20. $\frac{9000}{10000}, \frac{700}{10000}, \frac{30}{10000}, \frac{1}{10000}$. 21. $\frac{16}{24}, \frac{10}{24}, \frac{9}{24}$, or $\frac{2}{3}, \frac{5}{12}, \frac{3}{8}$.
22. $\frac{22}{36}, \frac{21}{36}, \frac{20}{36}$, or $\frac{11}{18}, \frac{7}{12}, \frac{5}{9}$. 23 $\frac{15}{30}, \frac{12}{30}, \frac{10}{30}, \frac{9}{30}$, or $\frac{1}{2}, \frac{2}{5}, \frac{1}{3}, \frac{3}{10}$.
24. $\frac{99}{154}, \frac{70}{154}, \frac{66}{154}$, or $\frac{9}{14}, \frac{5}{11}, \frac{3}{7}$. 25. $\frac{10}{12}, \frac{9}{12}, \frac{8}{12}$, or $\frac{5}{6}, \frac{3}{4}, \frac{2}{3}$.
26. $\frac{45}{105}, \frac{42}{105}, \frac{35}{105}$, or $\frac{3}{7}, \frac{2}{5}, \frac{1}{3}$.

EXERCISE XXXVII. (PAGE 113).

1. $1\frac{1}{12}$ 2. $2\frac{1}{4}$. 3. $\frac{11}{18}$. 4. $1\frac{11}{40}$. 5. $1\frac{7}{20}$. 6. $1\frac{6}{35}$.
7. $1\frac{47}{72}$. 8. $1\frac{7}{8}$. 9. 16. 10. $245\frac{23}{24}$. 11. $11\frac{17}{60}$.
12. $342\frac{13}{60}$. 13. $75\frac{59}{84}$. 14. $\frac{63}{64}$. 15. $3\frac{41}{81}$. 16. $1\frac{16}{35}$.
17. The sum of $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{6}$ is the greater by $\frac{2}{15}$. 18. Yes.
19. The first is the greater by $\frac{5}{72}$. 20. $522\frac{199}{210}$.

EXERCISE XXXVIII. (PAGE 116).

1. $\frac{1}{6}$. 2. $\frac{1}{12}$. 3. $\frac{2}{35}$. 4. $\frac{1}{12}$. 5. $7\frac{4}{7}$. 6. $\frac{7}{60}$.
7. $\frac{3}{7}$. 8. $3\frac{1}{6}$. 9. $6\frac{11}{18}$. 10. $3\frac{11}{36}$. 11. $2\frac{23}{30}$. 12. $15\frac{3}{8}$.

13. $4\frac{2}{9}$. 14. $11\frac{73}{75}$. 15. $\frac{9}{100}$. 16. $\frac{157}{252}$. 17. $22.50.
18. $\frac{1}{6}$. 19. $\frac{3}{5}$. 20. $\frac{1}{8}$.

EXERCISE XXXIX. (PAGE 119).

1. $3\frac{1}{3}$. 2. $\frac{3}{4}$. 3. 2. 4. $3\frac{3}{5}$. 5. $2\frac{1}{2}$. 6. 14.
7. $\frac{1}{6}$. 8. $\frac{5}{6}$. 9. $\frac{8}{15}$. 10. $7\frac{1}{2}$. 11. 24. 12. $22\frac{1}{2}$.
13. $\frac{1}{5}$. 14. $3\frac{3}{8}$. 15. $4\frac{1}{5}$. 16. $1\frac{1}{2}$. 17. $\frac{13}{15}$ 18. $\frac{1}{2}$.
19. $\frac{1}{4}$. 20. A has $\frac{42}{80}$, B has $\frac{45}{80}$. 21. $\frac{4}{27}$. 22. $\frac{3}{64}$.
23. $\frac{9}{125}$. 24. 75 cents. 25. A gets 55 cents, B $1.20.

EXERCISE XL. (PAGE 123).

1. $\frac{3}{14}$. 2. $\frac{7}{20}$. 3. $\frac{3}{8}$. 4. $\frac{3}{8}$. 5. $\frac{2}{3}$. 6. $1\frac{1}{3}$. 7. $\frac{3}{20}$.
8. $\frac{5}{12}$. 9. $\frac{1}{2}$. 10. $1\frac{8}{13}$. 11. $1\frac{11}{54}$. 12. $\frac{4}{45}$. 13. $1\frac{6}{7}$.
14. $\frac{4}{9}$. 15. $\frac{15}{28}$. 16. $\frac{1}{9}$. 17. $2\frac{2}{5}$ lbs. 18. $\frac{9}{10}$. 19. $\frac{9}{10}$.
20. $\frac{5}{9}$ of a day. 21. 6 days. 22. $12. 23. 75 marbles.
24. $\frac{5}{6}$.

EXERCISE XLI. (PAGE 126).

1. $\frac{11}{24}$. 2. $\frac{17}{24}$. 3. $\frac{29}{60}$. 4. $\frac{19}{36}$. 5. $\frac{41}{48}$. 6. $1\frac{11}{16}$.
7. $\frac{4}{15}$. 8. $\frac{7}{90}$. 9. $\frac{49}{90}$. 10. $\frac{13}{60}$. 11. $\frac{29}{30}$. 12. 5.
13. $1\frac{7}{8}$. 14. $22\frac{1}{2}$. 15. $\frac{2}{5}$. 16. $\frac{23}{55}$. 17. $2\frac{8}{15}$. 18. $\frac{3}{8}$.

EXERCISE XLII. (PAGE 128).

1. $1\frac{13}{15}$. 2. $\frac{5}{16}$. 3. $3\frac{1}{3}$. 4. $\frac{11}{19}$. 5. 1. 6. $50\frac{1}{7}$.
7. $\frac{13}{31}$. 8. $1\frac{61}{99}$. 9. $\frac{63}{247}$. 10. $\frac{3}{5}$. 11. $\frac{15}{16}$. 12. $8\frac{1}{3}$.

EXERCISE XLIII. (PAGE 129).

1. £7 16*s.* 6*d.* 2. £19 9*s.* 4$\frac{1}{2}$*d.* 3. £7 19*s* 11$\frac{1}{4}$*d.*
4. £20 14*s.* 2*d.* 5. 16*s.* 6*d.* 6. 5 ac. 4 per.
7. 2*f.* 22 per. 1 yd. 1 ft. 6 in. 8. £10 5*s.* 5*d.*
9. 1 lb. 3 oz. 16$\frac{7}{8}$ dwt. 10. 2 r. 29 per. 2 yds. 6 ft. 108 in.

11. 2 dys. 16 hrs. 10 mins. 12. $\$16.46\frac{2}{3}$. 13. $\$4.50$.
14. £5 12*s*. 15. $\$24.80$. 16. $\$21.50$.

EXERCISE XLIV. (PAGE 130).

1. $\frac{13}{20}$. 2. $\frac{4}{9}$. 3. $\frac{24}{55}$. 4. $\frac{151}{160}$. 5. $\frac{43}{96}$. 6. $\frac{144}{175}$.
7. $\frac{175}{192}$. 8. $\frac{7}{12}$. 9. $\frac{7}{8}$. 10. $\frac{1}{2}$. 11. $\frac{8}{17}$. 12. $\frac{127}{640}$.
13. $\frac{1}{6}$ of cash price or $\frac{1}{7}$ of price on credit. 14. $\frac{5}{16}$.
15. $\frac{810}{1109}$. 16. $\frac{3}{4}$. 17. $1\frac{5}{54}$. 18. $\frac{54}{59}$. 19. $24\frac{8}{13}$, $\frac{13}{320}$.

MISCELLANEOUS EXAMPLES,

PAPER I. (PAGE 131).

2. $1\frac{521}{1980}$. 3. 1260. 5. $\frac{13}{4}$. 6. $4\frac{37}{60}$.

PAPER II. (PAGE 132).

2. $\frac{17}{60}$. 3. It will be less than either of them. 4. $\frac{3}{7}$.
5. $23\frac{365}{411}$. 6. $20\frac{1}{40}$.

PAPER III. (PAGE 132).

1. $\frac{33}{64}$. 2. $7913\frac{2}{11}$ miles. 3. $\frac{51}{60}$, $\frac{50}{60}$, $\frac{40}{60}$, $\frac{36}{60}$, $\frac{35}{60}$, $\frac{2}{3}$.
4. 1. 5. $\frac{4336}{58881}$. 6. 104 days.

PAPER IV. (PAGE 133).

1. $\frac{240}{193}$ is the greater by $\frac{1}{171191}$. 2. $4\frac{13}{30}$. 3. $366\frac{8}{111}$, 366.
4. $\frac{139}{244}$. 5. $\$12$. 6. 1st gains $\$6\frac{2}{3}$ more than 2nd.

PAPER V. (PAGE 134).

1. $\frac{1}{5}$. 2. $6\frac{38}{231}$, $\frac{431}{489}$. 3. $33\frac{1}{3}$ cts., $\$40$. 4. $1\frac{1}{7}$.
5. $\$1200$. 6. 400000000 bushels. 7. $\frac{23}{77}$.
8. $\frac{3}{8}$, $\frac{45}{88}$, $\frac{25}{396}$, $\frac{5}{99}$. 9. 9600 acres.

EXERCISE XLV. (PAGE 137).

1. ·3, ·47, ·08, ·016, ·508. 2. $\frac{7}{10}$, $\frac{63}{100}$, $\frac{4}{100}$, $\frac{17}{1000}$, $\frac{9}{1000}$, $\frac{706}{1000}$. 3. 7·8, 61·35, 600·06, ·0023, 8·6, 9·65. 4. $5\frac{6}{10}$, $24\frac{1}{100}$, $40\frac{4}{10}$, $60\frac{6}{100}$, $2\frac{3456}{10000}$, $77\frac{77}{100}$. 5. ·6, ·78, ·009, 2·5, 0018, 000405. 6. 31·4159, 314·159, 3141·59. 7. 1·7308, ·17308, ·017308.

EXERCISE XLVI. (PAGE 138).

1 63 673. 2. 487·993. 3. 219·3008. 4. 868 2659. 5. 535·803. 6. 8·8888. 7. 145·283. 8. 654·789. 9. 671·012. 10. 160 0861. 11. 88888·88888. 12. 864·10247 13. 1·90625.

EXERCISE XLVII. (PAGE 140).

1. 9·186. 2. ·6412. 3. 11·036. 4. ·100125. 5. 7·8232. 6. ·07992. 7. 381·83. 8. 5·784. 9. ·0771. 10. ·785. 11. 76·86. 12. ·00001. 13. 2·5. 14. ·095. 15. 1·823. 16. 59·94. 17. 12·9026. 18. ·03125. 19. By ·00536. 20. ·29. 21. $6486·49. 22. A's $2325, B's $2675.

EXERCISE XLVIII. (PAGE 143).

1. 1·5. 2. ·2. 3. ·012. 4. 6·4. 5. ·0024. 6. 1·62. 7. 25. 8. ·00052. 9. 61·64. 10. 429·84 11. ·024. 12. 6·003. 13. 63·35. 14. ·093888. 15. 1998·136. 16. ·0040248. 17. $7.35. 18. 75 cents. 19. ·455 and ·245. 20. ·26. 21. $2608·956.

EXERCISE XLIX. (PAGE 146).

1. ·04. 2. ·4. 3. ·024. 4. 6·15. 5. ·13. 6. ·5. 7. ·09. 8. 3·75. 9. 4·8. 10. 1·35. 11. 135.

12. 13500. 13. 1700·1. 14. ·17001. 15. 18030. 16. ·327137114. 17. 75 cents. 18. 8 hours. 19. 180 pupils. 20. 25 hours.

EXERCISE L. (PAGE 147).

1. 5·433. 2. ·076. 3. ·134. 4. ·051. 5. ·716. 6. 2·868. 7. 6·233. 8. ·115. 9. 4·7313. 10. 1680·675 feet. 11. 12·086 metres. 12. $1.19. 13. 1·007557 pts. 14. ·007.

EXERCISE LI. (PAGE 150).

1. ·125. 2. ·375. 3. ·875. 4. ·6875. 5. ·65. 6. ·245. 7. 2·75. 8. 7·8. 9. ·96875. 10. 1·88. 11. ·0875. 12. ·0075. 13. ·667. 14. ·833. 15. ·857. 16. ·727. 17. ·583. 18. ·295. 19. ·471. 20. 1·316. 21. $\cdot\dot{1}$. 22. $\cdot\dot{5}$. 23. $\cdot\dot{1}\dot{7}$. 24. $61\cdot 1\dot{2}$. 25. $\cdot 00\dot{1}$. 26. $\cdot 29\dot{5}\dot{4}$. 27. $13\cdot\dot{5}7142\dot{8}$. 28. $9\cdot 2\dot{3}\dot{6}$. 29. $\cdot 3\dot{8}4615\dot{3}$. 30. $8\cdot\dot{0}5\dot{4}$. 31. $\cdot 98\dot{0}7692\dot{3}$. 32. $3\cdot 0\dot{6}1538\dot{4}$. 33. $\cdot 0\dot{0}5\dot{4}$. 34. $1\cdot 69\dot{7}2\dot{9}$. 36. $5\cdot\dot{0}7\dot{4}$. 36. $\cdot 0\dot{0}7\dot{4}$.

EXERCISE LII. (PAGE 151).

1. $\frac{1}{4}$. 2. $3\frac{1}{20}$. 3. $\frac{9}{40}$. 4. $\frac{7}{400}$. 5. $17\frac{3}{5}$. 6. $9\frac{7}{8}$. 7. $\frac{1}{125}$. 8. $7\frac{13}{20}$. 9. $\frac{8}{125}$. 10. $12\frac{7}{20}$. 11. $\frac{7}{16}$. 12. $4\frac{14}{25}$.

EXERCISE LIII. (PAGE 153).

1. $7\frac{2}{3}$. 2. $5\frac{8}{9}$. 3. 13. 4. $5\frac{1}{3}$. 5. $6\frac{8}{11}$. 6. $11\frac{1}{6}$. 7. $10\frac{1}{12}$. 8. $\frac{17}{36}$. 9. $\frac{26}{27}$. 10. $8\frac{5}{27}$. 11. $\frac{35}{74}$ 12. $\frac{63}{370}$. 13. $\frac{5}{7}$. 14. $\frac{76923}{250000}$. 15. $\frac{5}{26}$. 16. $\frac{3}{88}$. 17. By ·00119.

EXERCISE LIV. (PAGE 154).

1. $1{\cdot}3\dot{0}\dot{1}$. 2. $\cdot\dot{7}0\dot{3}$. 3. $\cdot412\dot{0}3\dot{7}$. 4. $\cdot\dot{2}$. 5. $5{\cdot}\dot{9}\dot{4}$. 6. $2{\cdot}541\dot{6}$. 7. $\cdot3\dot{3}\dot{6}$. 8. $9{\cdot}1\dot{6}$. 9. $\cdot\dot{4}9140\dot{0}$. 10. $\cdot07\dot{4}\dot{3}$. 11. ·24254. 12. ·02401. 13. 87·66771. 14. $\cdot000\dot{3}$. 15. $\cdot\dot{3}\dot{6}$ & 70 cts. 16. $\cdot\dot{1}\dot{8}$.

EXERCISE LV. (PAGE 156).

1. 15 cents. 2. $1,38. 3. $31.57½. 4. 3*s.* 9*d.* 5. £7 8*s.* 9*d.* 6. £35 14*s.* 4⅘*d.* 7. £1 10*s.* 8. 30 a. 38 per. 12 yds. 129·6 in. 9. 11 oz. 6 dwt. 12 grs. 10. £1 12*s.* 3*d.* 11. 21 hrs. 1 min. 26·4 sec. 12. 15 oz. 15 drs. 13. 13*s.* 4*d.* 14. £1 11*s.* 8*d.* 15. $36.00. 16. 6 fur. 14 per. 3 yds. 17. $39.42. 18 6*s.* 19. 3 ro. 21 per. 20. 3 c. yds. 4 c. ft. 21. 1 ro. 29 per. 18 yds. 1 ft. 50·4 in. 22. 10 min. 48 sec. 23. £2 1*s.* 11·2*d.* 24. 3 lbs. 1 oz. 10 dwt. 25. £1 4*s.* 6·768*d.* 26. 1 ro. 32·08 per.

EXERCISE LVI. (PAGE 158).

1. ·4375. 2. ·75. 3. ·8965. 4. ·375. 5. ·640625. 6. ·709375. 7. $\cdot1041\dot{6}$. 8. $\cdot\dot{6}$. 9. $\cdot49\dot{2}\dot{4}$. 10. ·9375. 11. $\cdot911458\dot{3}$. 12. ·7125. 13. ·5. 14. A, ·33 ; B, ·45 ; C, ·22. 15. 65·078125 c. ft. 16. $\dot{6}$. 17. 456·91906 grammes, and 375.97911 grammes. 18. 1·60935 kilometres and $4{\cdot}970\dot{9}\dot{5}$ furlong.

MISCELLANEOUS EXAMPLES.

PAPER I.

2. 1·3, 19880, ·03. 4. (a) $\frac{85}{1377}$, (b) $\frac{3}{11}$. 5. ·0020304 and 3000. 6. (a) ·300712, (b) $\cdot4\dot{4}6153\dot{8}$, $\frac{21}{80}$, $\frac{53}{88}$.

PAPER II.

1. (a) 5·068, (b) 50·68. 3. 1. 4. 1. 5. 63 ft. 6. $\cdot00556\dot{1}$.

PAPER III.

1. ·000002355. 2. ·00375, and $\cdot17\dot{8}5714\dot{2}$, and $\frac{17}{72}$. 3. 1155, 1716. 4. $75.60. 5. 5 hrs. 6. ·0348.

PAPER IV.

1. ·163, ·036, ·005868. 2. 7. 3. $12.35. 4. 60 cents. 5. $58.37.

PAPER V.

1. $\frac{3}{4}$, $\frac{1}{2000}$, ·017359. 2. ·00007. 4. ·4048. 5. ·00257. 6. $120. 7. 8 hrs. 46 min. 42 sec.

EXERCISE LVII. (PAGE 164).

1. $66·025. 2. $2587. 3. £10 6*s.* 9¾*d.* 4. £7 16*s.* 2¼*d.* 5. $5782. 6. £34 1*s.* 7. $2460.50. 8. $1146.60. 9. £1073 15*s.* 6*d.* 10. £477 15*s.* 11. £2666 16*s.* 6*d.* 12. $172.26. 13. $58.812. 14. 3560884·2875. 15. $95·904. 16. $222.739875. 17. $264·528. 18. £135 3*s.* 11¼*d.* 19. 206 m. 6 f. 20 p. 20. $908.47125. 21. £646 17*s.* 3⅜*d.* 22. £12635 12*s.* 3⅜*d.* 23. $657.9125.

EXERCISE LVIII. (PAGE 168).

1. $31.25. 2. $\$4.37\frac{1}{2}$. 3. $12422\frac{58}{161}$ b. 4. $\$1947\frac{7}{19}$.
5. $2.50. 6. 2 p. c. 7. $2800. 8. 50 cts. 9. $15.25.

EXERCISE LIX. (PAGE 170).

1. $43.50. 2. $63.91. 3. $\$21.98\frac{5}{8}$. 4. $\$547.63\frac{17}{25}$.
5. $258.30. 6. $235.008, $760.608. 7. $47.3645.
8. $\$90.13\frac{6}{73}$. $\$4.53\frac{6}{73}$. 9. $\$112.61\frac{23}{73}$.
10. £212 10*s.* 10*d.* nearly.

EXERCISE LX. (PAGE 172).

1. $14\frac{2}{7}$ yrs. 2. £50. 3. 7 p. c. 4. $544.41.
5. $25000.

EXERCISE LXI. (PAGE 173).

1. $784.87. $24.06. 3. $\$5208.33\frac{1}{3}$. 4. $669.15.

EXERCISE LXII. (PAGE 175).

1. 18. 2. 307. 3. 617. 4. 729. 5. 12·6. 6. 4·76.
7. 10·01. 8. 5·93. 9. ·359. 10. ·0907. 11. ·7008.
12. 2·236. 13. ·547. 14. ·173. 15. 2·469. 16. ·790.
17. 6·613. 18. 5·277. 19. 2·878. 20. 8·5042.

EXERCISE LXIII. (PAGE 177).

1. 4 : 3. 2. 4 : 3. 3. 3 : 5. 4. 3 : 4. 5. 8 : 1.
6. 4 : 5. 7. 5 : 7. 8. 9 : 10. 9. 5 : 9. 10. 3 : 4.
11. 5 : 7. 12. 25 : 16. 13. 5 : 6. 14. 7 : 8. 15. 15 : 8.
16. 144 : 175, 192 : 175. 17. 73 : 75. 18. 3 : 2.
19. A $28, B $32.

MISCELLANEOUS PROBLEMS. (PAGE 178).

2. \$590. 3. $\$31.63\frac{1}{2}$. 4. $2666.66\frac{2}{3}$. 5. 96 ac. 3 r. 30 p.
6. $\$5.46\frac{7}{8}$. 8. $92\frac{1}{4}$ grs. 9. \$312.50. 10. \$13650.
12. 128 men. 13. \$341.25. 14. 14 days. 16. \$766.
17. \$5.70. 18. $15\frac{5}{9}$ days. 20. $3\frac{2}{7}$ ac. 21. 20 ac.
22. \$9600. 23. \$6410. 24. 22c. 25. $247\frac{1}{2}$ ft. 26. \$450.
28. \$11.40. 29. 64 yds. 30. \$78.75. 31. \$80.75
33. 230400 bricks. 34. 172800 bricks. 35. $2346\frac{2}{3}$ yds.
36. \$560. 37. \$17.64. 38. $\$16\frac{8}{13}$. 39. 415·5 gal., nearly.
40. 2·72 oz., nearly. 42. $10\frac{10}{11}$ m. past 3. 43. $38\frac{2}{11}$ m.
past 2. 44. $32\frac{8}{11}$ m. past 3. 45. $16\frac{4}{11}$ m. past 6.
46. $16\frac{4}{11}$ m. past 9. 47. $38\frac{2}{11}$ m. past 7. 49. 24 m. ;
A 4 r., B 3 r. 50. $3\frac{3}{7}$ m. ; $\frac{4}{7}$ r. for A, $\frac{3}{7}$ r. for B.
51. $192\frac{1}{2}$ m. ; A $38\frac{1}{2}$ r., B $27\frac{1}{2}$ r., C $17\frac{1}{2}$ r. 52. 80 yds.
53. 21 m. 54. A 1100 yds., B 880 yds., C 660 yds.
55. 2160. 56. ·008. 57. A 27 h., B 36. 58. ·081.
60. ·175 ; 112·5. 61. 732 men. 62. 700. 63. $1\frac{93}{109}$.
64. Increased by $\frac{5}{63}$. 65. ·00322. 66. ·73, by $\frac{497}{416100}$.
67. $\frac{5}{8}$. 68. A 180 ac., B 90 ac., C 30 ac.
69. A's 42 yrs., B's 32 yrs. 70. $\$1.18\frac{14}{27}$.
72. $14\frac{2}{9}$ oz. 73. \$12. 74. $\frac{1}{132}$, $\frac{1}{5500000}$. 75. 74·6875 fl.
76. 8 lbs. 5 oz. 5 dwt. 77. 143·127 fr. 78. $\cdot 97\dot{3}$.
79. 116·75 ch. 80. £8 16*s*. $3\frac{3}{4}$*d*. 81. $\cdot\dot{5}1\dot{8}$, £3 7*s*. 6*d*.
82. £2 11*s*. 4·59*d*. 83. $1\frac{3}{5}$ p. 85. $\frac{40}{72}$, $\frac{63}{72}$, $\frac{54}{72}$.
86. $\frac{9}{12}$, $\frac{9}{12}$, $\frac{10}{12}$, $\frac{6}{12}$. 87. 24 h. 88. \$18000. 89. \$90.
90. $\frac{39}{1600}$ 91. ·03434. 92. A \$15, B \$13.50. 93. \$2400.
94. $\$320.65\frac{1}{4}$. 95. £78 6*s*. $11\frac{2}{5}$*d*., 78 ac. 1 r. $7\frac{4}{5}$ p., 78 lbs.
4 oz. 3 dwt. $9\frac{3}{5}$ grs. 96. $8\frac{8}{9}$ p. c. 97. 1 yr. $4\frac{2}{3}$ m.

98. ·2171875*d*., ·304 of 1 lb. 12 oz. 99. $\frac{7}{48}$. 100. 9 oz.
8 dwt. $\frac{1}{2}$ gr. 101. ·0222112. 102. 2.84544 lbs.
104. 19·5696 in. 105. 24 men. 106. 64. 107. 40 p.
108. $21. 109. $3.75, $4.80, $72. 110. $1992045.
111. $\frac{28}{23}$. 112. 375*d*. 113. 9 metres. 114. 50*d*.
115. $20\frac{5}{14}$ in. 116. $8\frac{1}{3}$. 117. 12 boys. 118. A $13.50,
B $12, C $9. 119. $8·16\frac{7}{13}$ fr. 120. 2 lbs. 1 oz. 14 dwt. 8grs.
121. 24 men. 122. $\frac{3325}{3432}$. 123. 1 ac. 3 r. 29 p. 1 yd.
5 ft. $139\frac{7}{23}$ in., 75 times. 124. $3\frac{7}{12}$, $2\frac{19}{48}$, $3\frac{49}{192}$.
125. 71·07868. 126. ·064, 46·875. 127. $4\frac{2}{3}$ m. from
Tor., 4 m. 128. $\frac{1333}{1000000}$. 129. 11547·398 fr.
130. 15 lbs. 3 oz. $9\frac{1}{7}$ dwt. 131. $\frac{5}{27}$, $\frac{19}{55}$, $\frac{263}{300}$. 132. ·42 of
stock. 133. $\frac{98}{280}$, $\frac{35}{280}$, $\frac{120}{280}$. 134. $9000. 135. $10\frac{2}{3}$ h.
136 ·875 g. 137. $\$6\frac{3}{22}$, $\$4\frac{10}{11}$. 138. 17 times, 1 lb.
4 oz. 13 dwt. 5 grs. 139. 17, $\frac{45}{52}$, $\frac{97}{39780}$. 140. £17 13*s*. 8*d*.
141. $1\frac{1}{20}$. 142. ·0405. 143. The units are 1*d*., 1*s*., £1 :
180*d*. = 15*s*. = £$\frac{3}{4}$. 144. 25c. 145. 9 m.
146. (1) $19.92, (2) $14.40. 147. $\frac{27}{44}$. 148. £10 6*s*.
149. $4600. 150. A $40, B $48. 151. $50.
152. $26\frac{91}{145}$ yds. 153. 8·79375. 154. $297.60, $2178.
155. 45 m. 156. 1 h. 24 m. 157. $918\frac{27}{32}$ tons.
158. $19\frac{3}{73}$c., 5·25 fr.

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